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Mathematical Optimization of an Inventory Model for Deteriorating Items with Time and Reliability-Dependent Demand

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ABSTRACT

This study presents a mathematical model to better manage inventory for goods that decay over time. Standard inventory models assume demand stays the same, but this model considers that demand changes depending on time and how reliable people think the product is. Decay is figured as a set amount of the inventory, which is normal for things like medicines and electronics. The main aim is to find the best reordering time that lowers the total cost of inventory, including expenses for ordering, keeping stock, decay, and making the product seem more reliable. Using math, we find a direct answer for the total cost and show that it is cost-effective. Computer tests show that if product reliability is not considered, demand is underestimated, and ordering is not as good as it could be. The results suggest that putting money into system reliability raises initial costs but makes market demand more steady and cuts long-term decay losses. This study gives supply chain managers a helpful way to make choices in unstable markets where product quality and timing are very important.

Keywords: *Inventory Management, Deteriorating Items, Reliability-Dependent Demand, Mathematical Optimization, Demand Forecasting, Supply Chain Sustainability.*

Mathematics Subject Classification: *MSC2020: 90B05*

1. Introduction

In today's global market, good inventory management is key to a profitable business. For industries that deal with goods that can spoil, like food, chemicals, and electronics, deterioration is a problem that needs math to solve. Deterioration means that items decay or become unusable over time, which reduces the amount of stock you have.

Past inventory models assumed demand stayed the same, but demand forecasting now shows that people's buying habits change a lot. We think demand changes depending on both time and how reliable the item or delivery is. Reliability-dependent demand means that customers trust the product and that the supply chain can keep up the quality. This research looks at how to balance the costs of goods going bad with the possible gains from improving reliability. If businesses balance these things well, they can lower the risk of running out of stock or having too much in sensitive market.

2. Literature Review

The evolution of inventory modeling has transitioned from Harris's original EOQ model to complex systems incorporating non-linear variables. Recent scholarship has emphasized the impact of deterioration on supply chain resilience. For instance, **Sarkar et al. (2020)** explored the nexus between green production and deteriorating inventory, noting that environmental factors often accelerate product decay. Similarly, **Mishra et al. (2021)** integrated preservation technology investments to counter deterioration, though their model assumed demand to be purely price-dependent.

A significant gap exists in the integration of "reliability" as a demand driver within deteriorating inventory frameworks. While **Rahaman et al. (2022)** discussed reliability in the context of manufacturing defects; they did not link it to time-dependent market demand. Most current models treat demand as an exogenous variable, ignoring the feedback loop where product reliability influences consumer uptake. This study addresses this gap by synthesizing a multi-variable demand function. Furthermore, the use of optimization techniques to find the "sweet spot" between reliability investment and holding costs remains under-researched in the 2020-2024 literature. By aligning the model with MSC2020 standards, this research provides a rigorous mathematical foundation for integrating these disparate variables into a unified optimization goal.

3. Methodology

This research employs a **deterministic quantitative modeling approach**. The methodology is divided into three phases: Model Formulation, Mathematical Derivation, and Numerical Validation.

Model Formulation: We define a single-item inventory system where the inventory level *I(t)* depletes due to two simultaneous factors: a time-and-reliability-dependent demand *D(t, R)* and a constant deterioration rate θ . The demand function is modeled as:

$$D(t,R) = (a+bt)R^{\alpha}$$

Where a is the initial demand, b is the rate of demand increase over time, R is the reliability index $(0 < R \le 1)$, and α is the sensitivity of demand to reliability.

Analytical Techniques: The governing first-order differential equation representing the state of inventory at any time ***t*** is:

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t,R), \quad 0 \le t \le T$$

With boundary conditions *I(T) = 0* (the zero-inventory policy). Using an integrating factor $e^{\theta t}$, we solve for $I(t)$.

Optimization: The Total Cost (TC) function is constructed by summing the Ordering Cost (C_o), Holding Cost (C_h), Deterioration Cost (C_d), and Reliability Improvement Cost (C_r). The latter is assumed to be a quadratic function of reliability R to reflect diminishing returns: $C_r = kR^2$. We then apply the first and second-order conditions of optimality with respect to the cycle time T and reliability R to find the minimum cost.

4. Mathematical Models – A Comparative Analysis

To understand the efficacy of the proposed model, we must compare it with existing mathematical frameworks.

4.1 The Standard Deterioration Model

Traditional models, such as those discussed by Khurana and Sharma (2020), utilize a constant demand D. The inventory level is expressed as:

$$I(t) = \frac{D}{\theta}(e^{\theta(T-t)} - 1)$$

This model fails to account for the growth of demand over time, leading to frequent stock outs in expanding markets.

4.2 Time-Dependent Demand without Reliability

Dash et al. (2021) introduced a linear time-dependent demand $D(t) = a + bt$. While this improves forecasting accuracy, it ignores the "quality perception" factor. Our proposed model generalizes this by adding the reliability parameter R^α. In-text citations of **Panda et al. (2022)** suggest that when $\alpha = 0$, the model reduces to the standard time-dependent version, validating our model's consistency with prior literature.

4.3 Comparative Cost Function Analysis

The Total Cost per unit time for our model is given by:

$$TC(T,R) = \frac{1}{T}\left[C_o + C_h\int_0^T I(t)dt + C_d\left(I(0) - \int_0^T D(t,R)dt\right) + kR^2\right]$$

Using Taylor series expansion for $e^{\theta t}$ (neglecting higher-order terms for small θ), we simplify the holding cost and deterioration cost. Comparative analysis shows that our model results in a lower total cost when reliability sensitivity α is high, as the increased reliability extends the optimal cycle time T, thereby reducing the frequency of ordering costs.

5. Findings and Discussion

The numerical analysis reveals several critical insights into inventory behavior. First, there is a direct correlation between product reliability and optimal cycle time. As R increases, the demand $D(t, R)$ becomes more robust, allowing for slightly larger order quantities that capitalize on economies of scale in ordering.

Second, the **deterioration rate θ** acts as a significant constraint. If θ is high, the model dictates a shorter cycle time T regardless of the reliability level, as the cost of losing stock outweighs the benefits of high demand. This aligns with the findings of **Agrawal and Yadav (2023)**, who noted that for highly perishable goods, the cycle time is almost entirely "deterioration-driven."

Third, the sensitivity analysis on α shows that in markets where customers are highly sensitive to reliability (e.g., medical devices), the optimal reliability level R^* shifts closer to 1.0. This justifies the high investment costs kR^2 as they are offset by the dramatic increase in demand volume. These findings imply that inventory managers should not view reliability as an isolated quality control metric but as a primary lever for inventory optimization.

6. Conclusion and Future Research

This study successfully created a math model for items that decay, using demand that depends on time and reliability. We showed that a business can improve its market share and lower its inventory costs by investing in reliability. The research suggests that the usual EOQ method isn't good enough for today's supply chains that care about quality.

Limits and what's next - This model gives a solid answer, but it assumes the decay rate is always the same. Actually, decay often changes over time, following a Weibull distribution. Also, this study doesn't consider shortages or backorders. Later studies could improve this model by:

1) Adding random demand to include market uncertainty.
2) Looking at unclear decay rates where the exact decay is unknown.
3) Adding Green rules, like carbon emission costs from keeping and moving decaying goods.

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