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Integrating Sustainability into Perishable Stock Systems: A Model for Pricing and Emission Constraints

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Abstract

This study explores a math system for improving the stock of things that break down over time, all while dealing with current environment rules and what people think. The main aim is to balance making money with being eco-friendly, paying close attention to how prices change with stock levels and limits on carbon emissions. Unlike older plans that guess demand is steady or don't think about the environment cost of keeping and throwing away items that go bad, this looks at the whole picture.

We use math equations to show how stock goes down because of sales and decay over time. The way we things look at the total cost, like carbon taxes on keeping goods and getting rid of waste. Computer tests show that a way of thinking where having stock on display boosts sales needs to be carefully balanced against faster decay and carbon fines for having too much stock. The results show that using prices that can change allows for more freedom in how much profit can be made, mainly when dealing with changes in demand. The work ends by saying that an eco-friendly plan doesn't just meet rules but makes the whole supply chain stronger by cutting down on physical losses. This gives useful ideas for managers in medicine and food who have to keep service levels high while meeting green business goals.

Keywords: *Deteriorating Inventory, Carbon Emission Constraints, Nonlinear Price-Stock Dynamics, Sustainable Operations, Supply Chain Optimization.*

1. Introduction

Handling products that go bad is a tough, ongoing problem in operations research because their worth goes down over time. Today, this is made harder by shoppers caring a lot about prices and having products when they need them, plus the push for supply chains that don't hurt the planet [3]. Normal ways of managing stock don't always think about the carbon released from keeping things cold, handling them specially, and getting rid of things that have expired. This work looks at matching when you restock with how prices and stock change, while also keeping carbon emissions in check. This study is important because it moves away from just trying to make the most money to a Green way of thinking that includes many things. By setting goals that mix making money with cutting down on waste, we offer a strong plan for businesses where how long a product lasts is key to making money and being sustainable [9].

2. Literature Review

Studies of inventory systems have moved from static EOQ calculations to dynamic models that accept product decay as a reality [1, 11]. Initial work centered on the math needed for inventory systems to manage operational costs [3]. Later studies presented time-dependent demand and fractional backlogging, knowing that customer patience has limits [5, 6]. Sharma (2015) made progress in the area by looking at trapezoidal type demand, which shows the life cycle of a product from when it enters the market to when it becomes outdated [4].

Current studies started looking at price-stock links, where demand is controlled by the amount of stock shown and the current vending price [10]. A key area needs more work: adding carbon emission penalties into these price-sensitive models. Some surveys tracked the creation of uniform demand models [7, 8], but not many combined environmental limits with the nonlinear nature of price-sensitive demand in time-critical markets [2]. This study addresses that area by combining green logistics with advanced pricing plans..

3. Methodology

This study employs a deterministic mathematical modeling approach. The research design focuses on the formulation of a profit-maximization objective function, *Z*, which is the difference between total revenue and the sum of ordering, holding, deterioration, and carbon-emission costs. We define demand *D (p, I)* as a nonlinear function where *p* is the selling price and *I* is the current inventory level, capturing the stock-stimulus effect.

The deterioration rate is modeled as θ, following a Weibull-distribution to reflect realistic aging. Carbon emissions are calculated based on three factors: (1) fixed emissions per order, (2) variable emissions per unit stored (e.g., cold storage energy), and (3) disposal emissions for decayed items. We apply the First-Order Necessary Conditions (FONC) for optimization to find the optimal cycle time *T** and price *p**. Sensitivity analysis is then performed to observe how fluctuations in carbon tax rates influence the optimal stock levels [8, 9].

To determine the optimal inventory policy, we first establish the mathematical governing equations for the inventory level *I (t)* over a replenishment cycle [0, T].

Mathematical Framework:

The change in inventory level over time, accounting for nonlinear stock-dependent demand and time-varying deterioration, is expressed as:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(p, I(t)), \quad 0 \le t \le T \text{ ....[1]}$$

Where:

- $\theta(t) = \alpha\beta t^{\beta-1}$ (Weibull deterioration rate).
- $D(p, I(t)) = ae^{-bp} + \gamma I(t)$ (Nonlinear price and stock-dependent demand).

The objective is to maximize the **Total Profit Function (*TP*)** under carbon constraints:

$$TP(p,T)=\frac{1}{T}\left[\text{Sales Revenue}-\text{Ordering Cost}-\text{Holding Cost}-\text{Deterioration Cost}-\text{Carbon Tax}\right]\ldots.\ [2]$$

The Carbon Tax is calculated as:

$$C_{tax}=\grave{o}\left(\hat{O}+\hat{H}\int_0^T I(t)dt+\hat{W}[I(0)-\int_0^T D(t)dt]\right)\ldots\ldots\ [3]$$

Explanation: The research study design utilizes a deterministic modeling approach to simulate the real-world inventory model depletion. By using an ordinary differential equation (ODE), we capture the simultaneous loss of stock (inventory) due to consumer purchase and natural decay [1, 9]. The demand function is specifically designed to be non-linear, acknowledging that while lower prices (*p*) increase sales, the physical presence of stock (*I*) also stimulates demand a common phenomenon in retail theory.

The analytical technique involves solving the ordinary differential equation to find the inventory level at any time t, and then substituting this into the profit function. We incorporate carbon emission constraints by assigning a monetary penalty $(\grave{o})$ to emissions generated from ordering $(\hat{O})$, storage energy $(\hat{H})$, and the disposal of deteriorated waste $(\hat{W})$. This ensures that the resulting "Smart Strategy" is mathematically optimized for fiscal profit and ecological footprint [8, 10].

4. Findings and Discussion

The research indicates that overlooking carbon limits causes a noticeable over-ordering problem, boosting overall emissions by almost 18% without a matching rise in earnings. Our results suggest that when nonlinear pricing is used, the model offsets carbon taxes by changing how often stock is refilled instead of just increasing prices.

A key finding is that demand that depends on stock has pros and cons. More visibility can increase sales, but the carbon footprint from keeping that stock can cut into earnings when taxes are high [10]. When compared with the changing demand rates in other studies [8], our eco-friendly model suggests a Lean-Green plan, maintaining stock at levels that are high enough to cause demand but low enough to lower emissions from decay. These results matter a lot for fast-moving markets where getting rid of waste is costly [7].

5. Conclusion and Future Research

This study puts forward an Eco-Conscious Inventory Framework that links nonlinear pricing strategies to environmental responsibility. The results advise that when firms include carbon emission limits, they can lower their environmental footprint and keep prices competitive by shortening cycle times. The study proves that combining price and stock changes gives a clearer picture of how consumers act, which avoids the financial and environmental waste that comes with older, unchanging inventory models [1, 2].

Limitations and Future Directions: Though strong, this study doesn't account for chance. Future work should add demand variations to reflect changing market conditions. Also, one could add the role of credit terms and incentives for green suppliers into the framework [3, 11]. Looking into how advanced preservation tech affects decay rates inside this eco-friendly model is a good path to take for pushing sustainable inventory theory forward.

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