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Application of Input–Output Analysis in the Indian Food Processing Industry: A Case Study Approach

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Abstract

This work explores analytical solutions of fractional-order dispersive partial differential equations using a newly proposed decomposition approach. Fractional derivatives are formulated in the Caputo sense. The obtained solutions, for both fractional and classical integer-order cases, are expressed as convergent series, highlighting the effectiveness and fast convergence of the technique. To demonstrate the reliability and applicability of the method, several representative examples are discussed in detail. Furthermore, the study examines how the solutions of fractional-order models gradually approach the corresponding integer-order solutions as the fractional parameter varies.

Keywords: *Nonlocal partial differential equations (FPDEs), Dispersive equations, Caputo Nonlocal derivative, Convergence analysis, Laplace transform.*

Mathematics Subject Classification (MSC) 2020: *35R11; 35Q53; 26A33; 35C10; 44A10.*

1. Introduction:

Partial differential equations (PDEs) are essential mathematical models used to represent a broad spectrum of physical processes in applied scientific disciplines, including fluid motion, biological modelling, quantum systems, reaction kinetics, and optical wave propagation. In 1895, Diederik Korteweg and Gustav de Vries formulated a dimensionless version of a relation that is now recognized as the KdV equation. This formulation is widely utilized for investigating dispersive wave dynamics in various branches of science and engineering, with particular relevance in plasma physics and quantum-mechanical contexts. Since exact analytical solutions of the KdV equation are typically difficult to derive, a variety of analytical methodologies have been proposed to construct approximate solutions. The KdV-type equations mainly consist of two significant dispersive components of third and fifth order. Notably, the 5th-order KdV equation [4] is of considerable importance in plasma-related studies. Computational and numerical analyses of both 3rd and 5th-order dispersive KdV models have also been reported in earlier literature.

The nonlinear behaviour of physical systems plays a crucial role in understanding their overall dynamics, highlighting the significance of nonlinear terms in mathematical models of physical phenomena. In this regard, reductive perturbation techniques for nonlinear KdV equations were examined in earlier studies. A variational approach was also proposed to obtain exact solutions of KdV equations involving higher-order nonlinearities. In addition, computational solutions of the KdV–Burgers equation were successfully achieved through the compact constrained interpolation framework using interpolation profiles

Furthermore, numerical investigations of fifth-order KdV equations were carried out using the homotopy perturbation transform method, while both third- and fifth-order KdV equations were analysed through different numerical approaches. Several studies have also explored entropy concepts in fractional calculus, entropy analysis for integer and fractional dynamical systems, nonlinear partial differential equations in entropy and convexity theory, and fractional derivative advection–diffusion models in two-dimensional semiconductor systems as well as national soccer league dynamics.

This method incorporates the advantages of the Adomian Decomposition Method with the Laplace transform to obtain accurate analytical solutions. Unlike numerical schemes such as the Runge–Kutta method, LADM does not require predefined step sizes, which makes it highly suitable for nonlinear mathematical models. In comparison with several other analytical procedures, LADM involves fewer computational parameters and avoids both discretization and linearization processes, enhancing its efficiency and simplicity.

A comparative study between LADM and the Adomian's Decomposition Method (ADM) for Nonlocal differential equations has been reported in previous research. The technique has also been successfully implemented for nonlinear partial differential framework arising in quantum field theory, particularly the Kundu–Eckhaus equation. In addition, the multi-steps Laplace Adomian decomposition approach has been developed for nonlinear fractional differential framework, while smoke model analysis has also been effectively investigated through LADM. Motivated by these studies, the present work aims to derive rigorous solutions of dispersive fractional calculus-based partial differential framework using the LADM's technique. The obtained solutions demonstrate strong in accordance with the true solution results and confirm the reliability and performance of the proposed technique.

2. Core Ideas and Preliminary Definition's

The present part presents various fundamental concepts and essential definitions related to fractional calculus. Various well-known definitions introduced by researchers including Riemann–Liouville's, Grünwald–Letnikov, together with Caputo are briefly discussed. These introductory explanations are included to provide the necessary background and understanding required for the present study.

Definition 1: Fractional integral defined in the Riemann–Liouville sense

$$\mathrm{I}_q^{\alpha} f(q) = \left\{ \begin{matrix} f(q) & if\alpha = 0 \\ \frac{1}{\Gamma\alpha}\int_0^y (q-\tau)^{\alpha-1} f(\tau)d\tau & if\alpha > 0 \end{matrix} \right\}$$

In this work, Caputo and collaborators [12] introduced a modified fractional derivative operator to address certain inconsistencies associated with the Riemann–Liouville derivative [5, 12]. This revised formulation, known as the Caputo fractional derivative, is particularly useful because it allows the use of Standard governing conditions are imposed for both fractional-order and integer-order differential equations.

Definition 2: Fractional derivative operator defined in the Caputo sense

The Caputo fractional derivative of order α is defined for $n \in N, y > 0, and\ f \in C^t(with\ t \geq -1)$. It is expressed as:

$$D^{\alpha} f(y) = \frac{\partial^{\alpha} f(y)}{\partial t^{\alpha}}.$$

and is given in piecewise form as:

- If $n' - 1 < \alpha \leq n', n' \in N$,then Caputo derivative is defined using an integral operator applied to the integer-order derivative of the function.
- For integer order cases, it reduces to the standard derivative of corresponding order.

In other words, the Caputo operator generalizes the usual differentiation to non-integer (fractional) order while still maintaining compatibility with classical derivatives when α becomes an integer.

Definition 3: Laplace Transform

Transformation of a function$g'(t)$, using the Laplace operator where$t > 0$, is a mathematical tool that converts a time-domain function into an p-domain representation. It is expressed as:

$$G'(u) = L\{g'(t)\} = \int_0^{\infty} e^{-ut} g'(t)\, dt$$

This transform is extensively utilized to facilitate the solution of differential equations by transforming them into algebraic forms, making the analysis and computation comparatively more convenient.

Definition 4: The convolution operation involving two functions

The Laplace transform of the convolution of functions $\boldsymbol{h_1(t')}$ and $\boldsymbol{h_2(t')}$ is expressed as

$$L[h_1 * h_2] = L[h_1(t')]L[h_2(t')],$$

where the symbol $*$ denotes the combined effect (convolution) operation. Convolution between the two functions is defined by

$$(h_1 * h_2)(t) = \int_0^t h_1(l)\, h_2(t-l)dl,$$

For a fractional derivative the combined effect (convolution), the Laplace transform is given by

$$L[D_t^\alpha g(t)] = u^\alpha G(u) - \sum_{i=0}^{m-1} u^{\alpha-1-i} g^i(0), \qquad m-1 < \alpha < m,$$

where $G(u)$ represents the Laplace image of h(t) and $h^i(0)$ denotes the values at the beginning of the integer-order derivatives of g(t).

3. Concept of the Fractional-order Laplace & Adomian decomposition technique

3.1 For One-Dimensional wave-spreading Equations

In this section, the decomposition method based on Laplace and Adomian concepts is employed to derive analytical solutions for third-order fractional partial differential equations exhibiting dispersive behaviour.

Consider the general Nonlocal dispersive equation

$$\frac{\partial^\alpha w(y,t)}{\partial t^\alpha} + v\frac{\partial^3 w(y,t)}{\partial y^3} = s(y,t),$$

Where

$$v, t \geq 0, \quad l-1 < \alpha < l \text{ and } \mathrm{s(y,t)} \text{denotes the source term.} \tag{1}$$

The problem is supplemented with the initial condition

$$w(y,0) = m(y),$$

boundary conditions are

$$w(0,t) = h_0(t), \;\; w_y(0,t) = h_1(t), \quad w_{yy}(0,t) = h_2(t). \tag{2}$$

Applying the Laplace-transform with respect to t, the governing equation becomes

$$L\left[\frac{\partial^\alpha w(y,t)}{\partial t^\alpha}\right] + L\left[v\frac{\partial^3 w(y,t)}{\partial y^3}\right] = L[s(y,t)], \tag{3}$$

By applying the Laplace-transform property for fractional-order derivatives defined by Caputo, we obtain.

$$L[w(y,t)] = \frac{m(y)}{\mathrm{u}} + \frac{1}{\mathrm{u}^\alpha} L[\mathrm{s(y,t)}] - \frac{1}{\mathrm{u}^\alpha} L\left[\mathrm{v}\frac{\partial^3 \mathrm{w(y,t)}}{\partial \mathrm{y}^3}\right] \tag{4}$$

According to the Adomian-decomposition approach, the unknown solution is expressed as an infinite series:

$$w(y,t) = \sum_{k=0}^\infty w_k\,(y,t). \tag{5}$$

If nonlinear terms are present, they are represented using Adomian polynomials:

$$Aw(y,t) = \sum_{k=0}^\infty A_k, \tag{6}$$

where the Adomian polynomial A_k is defined by

$$A_k = \frac{1}{k!}\frac{d^k}{d\alpha^k}\left[A\left(\sum_{k=0}^\infty \alpha^k w_k\right)\right]_{\alpha=0}, \qquad \text{k=0,1,2,....} \tag{7}$$

Substituting the decomposition series into the transformed equation yields

$$L\left[\sum_{k=0}^\infty w_k\,(y,t)\right] = \frac{\mathrm{m(y)}}{\mathrm{u}} + \frac{1}{u^\alpha} L[\mathrm{s(y,t)}] - \frac{1}{u^\alpha} L\left[\mathrm{v}\frac{\partial^3 \mathrm{w}_k(\mathrm{y,t})}{\partial \mathrm{y}^3}\right] \tag{8}$$

By utilizing the linearity of the Laplace-transform, the successive terms in the recursive form are derived as follows.

$$L[w_0(y,t)] = \frac{w(y,0)}{u} + \frac{1}{\mathrm{u}^\alpha} \mathrm{L[s(y,t)]},$$

$$L[w_1(y,t)] = -\frac{1}{u^\alpha} L\left[v\frac{\partial^3 w_0(y,t)}{\partial y^3}\right]$$

In general, the recurrence relation is

$$L[w_{k+1}(y,t)] = -\frac{1}{\mathrm{u}^\alpha} L\left[\mathrm{v}\frac{\partial^3 \mathrm{w_k(y,t)}}{\partial \mathrm{y}^3}\right], \;\; \mathrm{k} \geq 1. \tag{9}$$

Finally, By applying the Laplace inverse operator, the required result is obtained in the time domain.

$$w_0(y,t) = m(y,t),$$

$$w_{k+1}(y,t) = -L^{-1}\left[\frac{1}{u^{\alpha}} L\left[\mathrm{v}\frac{\partial^3 \mathrm{w_k(y,t)}}{\partial \mathrm{y}^3}\right]\right]. \tag{10}$$

These recursive relations generate successive approximations that converge to the analytical solution of the fractional dispersive equation.

3.2 Application of LADM to Higher-Dimensional Dispersive Equation's

Suppose the higher-dimensional Nonlocal dispersive PDE

$$\frac{\partial^{\alpha} w}{\partial t^{\alpha}} + d\frac{\partial^3 w}{\partial p^3} + e\frac{\partial^3 w}{\partial q^3} + h\frac{\partial^3 w}{\partial r^3} = s(p,q,r,t), \tag{11}$$

Where d, e, and h are constant coefficients, s(p,q,r) is the forcing factor, and fractional order satisfies

$$l-1 < \alpha < l, \qquad t \geq 0,$$

The corresponding Starting condition

$$w(p,q,r,0) = m(p,q,r).$$

The given fractional differential equation is solved by transforming it through the Laplace technique in terms of the time variable t, which converts the original equation into a simpler algebraic form in the transformed domain.

$$L\left[\frac{\partial^{\alpha} w}{\partial t^{\alpha}}\right] + L\left[d\frac{\partial^3 w}{\partial p^3} + e\frac{\partial^3 w}{\partial q^3} + h\frac{\partial^3 w}{\partial r^3}\right] = L[s(p,q,r,t)]. \tag{12}$$

Using the Laplace-transform formula for Caputo-fractional derivatives, we obtain

$$L[w(\mathrm{p,q,r},t)] = \frac{m(\mathrm{p,q,r})}{u} + \frac{1}{u^{\alpha}} L[\mathrm{s(p,q,r,t)}] - \frac{1}{u^{\alpha}} L\left[\mathrm{d}\frac{\partial^3 \mathrm{w}}{\partial \mathrm{p}^3} + \mathrm{e}\frac{\partial^3 \mathrm{w}}{\partial \mathrm{q}^3} + \mathrm{f}\frac{\partial^3 \mathrm{w}}{\partial \mathrm{r}^3}\right] \tag{13}$$

In Laplace Adomian Decomposition Method, the unknown function represented by an infinite series:

$$w(p,q,r,t) = \sum_{k=0}^{\infty} w_k\,(p,q,r,t) \tag{14}$$

Substituting expansion into transformed equation gives

$$L\left[\sum_{k=0}^{\infty} w_k\,(p,q,r,t)\right] = \frac{\mathrm{m(p,q,r)}}{\mathrm{u}} + \frac{\mathbf{1}}{\mathrm{u}^{\alpha}} L[\mathrm{s(p,q,r,t)}] - \frac{1}{\mathrm{u}^{\alpha}}\mathrm{L}\left[d\frac{\partial^3 w}{\partial p^3} + e\frac{\partial^3 w}{\partial q^3} + f\frac{\partial^3 w}{\partial r^3}\right] \tag{15}$$

The decomposition process begins with the zeroth component

$$L[w_0(p,q,r,t)] = \frac{w(p,q,r,0)}{u} + \frac{\mathbf{1}}{u^{\alpha}}\mathrm{L[s(p,q,r,t)]}$$

The next component is determined from

$$L[w_1(p,q,r,t)] = -\frac{1}{\mathrm{u}^{\alpha}}\mathrm{L}\left[d\frac{\partial^3 w_0}{\partial p^3} + e\frac{\partial^3 w_0}{\partial q^3} + f\frac{\partial^3 w_0}{\partial r^3}\right].$$

More generally, the recursive relation is written as

$$L[w_{k+1}(p,q,r,t)] = -\frac{\mathbf{1}}{u^{\alpha}}\mathrm{L}\left[d\frac{\partial^3 w_0}{\partial p^3} + e\frac{\partial^3 w_0}{\partial q^3} + f\frac{\partial^3 w_0}{\partial r^3}\right], \quad \mathrm{k} \geq 0 \tag{16}$$

Taking the back transformation from the Laplace domain leads to

$$w_0(p,q,r,t) = m(p,q,r,t),$$

$$w_{k+1}(p,q,r,t) = -L^{-1}\left[\frac{1}{\mathrm{u}^{\alpha}}\mathrm{L}\left(d\frac{\partial^3 w_0}{\partial p^3} + e\frac{\partial^3 w_0}{\partial q^3} + f\frac{\partial^3 w_0}{\partial r^3}\right)\right], \quad \mathrm{k} \geq 0 \tag{17}$$

Hence, the complete solution is obtained as a convergent series formed by successive approximations.

Illustration 1: Let us examine the fractional wave-dispersing Korteweg–de Vries model expressed in the following form [24].

$$\frac{\partial^{\alpha} w}{\partial t^{\alpha}} + 2\frac{\partial w}{\partial p} + \frac{\partial^3 w}{\partial p^3} = 0, \quad 0 < \alpha \leq 1 \tag{18}$$

subject to the starting state:

$$w(p,0) = cosp$$

By applying the Laplace-transform to the governing equation, the transformed expression in the Laplace-space is obtained as:

$$L\left[\frac{\partial^{\alpha} w}{\partial t^{\alpha}}\right] = -L\left[2\frac{\partial w}{\partial p} + \frac{\partial^3 w}{\partial p^3}\right] \tag{19}$$

Employing the Laplace-domain characteristic of the Caputo fractional operator:

$$u^{\alpha} L[w(y.t)] - u^{\alpha-1} w(y,0) = -L\left[2\frac{\partial w}{\partial p} + \frac{\partial^3 w}{\partial p^3}\right] \tag{20}$$

Applying the inverse Laplace operation, the solution in the time domain is obtained as:

$$w(p.t) = w(p.0) - L^{-1}\left[\frac{1}{\Gamma\alpha} L\left(2\frac{\partial w}{\partial p} + \frac{\partial^3 w}{\partial p^3}\right)\right] \tag{21}$$

Substituting the initial condition:

$$w(p.t) = cosy - L^{-1}\left[\frac{1}{\Gamma\alpha}L\left(2\frac{\partial w}{\partial p}+\frac{\partial^3 w}{\partial p^3}\right)\right]$$

By employing the decomposition technique of Adomian, the solution is constructed as follows.

$$w(p,t) = \sum_{k=0}^{\infty} w_k\,(p,t) \tag{22}$$

Where $\qquad w(p,0) = cosp$

and the recursive relation is:

$$w_{k+1}(p,t) = -L^{-1}\left[\frac{1}{\Gamma\alpha}L\left(2\sum_{k=0}^{\infty}\frac{\partial w_k}{\partial p}+\sum_{k=0}^{\infty}\frac{\partial^3 w_k}{\partial p^3}\right)\right] \tag{23}$$

The first few components are obtained as:

$$w_1(p,t) = sinp\frac{t^{\alpha}}{\Gamma\alpha+1}$$

$$w_2(p,t) = -cosp\frac{t^{2\alpha}}{\Gamma 2\alpha+1}$$

$$w_3(p,t) = -sinp\frac{t^{3\alpha}}{\Gamma 3\alpha+1}$$

Thus, the LADM solution becomes a series form:

$$w(p,t) = cosp + sinp\frac{t^{\alpha}}{\Gamma\alpha+1} - cosp\frac{t^{2\alpha}}{\Gamma 2\alpha+1} - sinp\frac{t^{3\alpha}}{\Gamma 3\alpha+1} + \cdots.$$

This can also be written as:

$$w(p,t) = cosp\left(1-\frac{t^{2\alpha}}{\Gamma 2\alpha+1}+\frac{t^{4\alpha}}{\Gamma 4\alpha+1}+\cdots\right)+sinp\left(\frac{t^{\alpha}}{\Gamma\alpha+1}-\frac{t^{3\alpha}}{\Gamma 3\alpha+1}+\cdots.\right) \tag{24}$$

Finally, when $\alpha = 1$, the solution reduces to the classical form:

$$w(p,t) = \cos(p-t). \tag{25}$$

Example 2: For the fractional dispersive KdV equation

$$\frac{\partial^{\alpha} w}{\partial t^{\alpha}}+\frac{\partial^3 w}{\partial p^3}+\frac{\partial^3 w}{\partial q^3} = 0, \qquad t > 0, \quad 0 < \alpha \le 1 \tag{26}$$

Along with the modified starting value condition

$$w(p,q,0) = \sin(p+q),$$

The problem is tackled by implementing the Laplace Adomian Decomposition Method.

After taking the Laplace-domain transformation and applying the inverse transform, the recursive relation becomes

$$w(p,q,t) = \sin(p+q) - L^{-1}\left[\frac{1}{u^{\alpha}}L\left(\frac{\partial^3 w}{\partial p^3}+\frac{\partial^3 w}{\partial q^3}\right)\right]. \tag{27}$$

Using the decomposition

$$w(p,q,t) = \sum_{k=0}^{\infty} w_k\,(p,q,t) \tag{28}$$

Components are obtained as follows.

Zeroth component

$$w_0(p,q,0) = \sin(p+q), \tag{29}$$

By ADM method

$$\sum_{k=0}^{\infty} w_k\,(p,q,t) = \sin(p+q) - L^{-1}\left[\frac{1}{u^{\alpha}}L\left(\sum_{k=0}^{\infty}\frac{\partial^3 w_k}{\partial p^3}+\sum_{k=0}^{\infty}\frac{\partial^3 w_k}{\partial q^3}\right)\right] \tag{30}$$

$$w_{k+1}(p,q,t) = -L^{-1}\left[\frac{1}{u^{\alpha}}L\left(\sum_{k=0}^{\infty}\frac{\partial^3 w_k}{\partial p^3}+\sum_{k=0}^{\infty}\frac{\partial^3 w_k}{\partial q^3}\right)\right] \tag{31}$$

First component

$$w_1(p,q,t) = -L^{-1}\left[\frac{1}{u^{\alpha}}L\left(\frac{\partial^3 w_0}{\partial p^3}+\frac{\partial^3 w_0}{\partial q^3}\right)\right]$$

Since $\qquad \frac{\partial^3}{\partial p^3}\sin(p+q) = -cos(p+q),$

$$\frac{\partial^3}{\partial q^3}\sin(p+q) = -cos(p+q),$$

We get

$$w_1(p,q,t) = 2\cos\,(p+q)\frac{t^{\alpha}}{\Gamma\alpha+1},$$

$$w_2(p,q,t) = -4\sin(p+q)\frac{t^{2\alpha}}{\Gamma 2\alpha+1}$$

Third component

$$w_3(p,q,t) = -8\cos(p+q)\frac{t^{3\alpha}}{\Gamma 3\alpha+1}$$

Hence the LADM series solution is

$$w(p,q,t) = \sin(p+q) + 2\cos(p+q)\frac{t^{\alpha}}{\Gamma\alpha+1} - 4\sin(p+q)\frac{t^{2\alpha}}{\Gamma 2\alpha+1} - 8\cos(p+q)\frac{t^{3\alpha}}{\Gamma 3\alpha+1} + \cdots. \quad (32)$$

For $\alpha = 1$, the series reduces to the precise solution

$$w(p,q,t) = \sin(p+q+2t). \quad (33)$$

Example 3: We consider the inhomogeneous nonlocal dispersive Korteweg–de Vries equation in 3D space.

$$\frac{\partial^{\alpha} w}{\partial t^{\alpha}} + \frac{\partial^3 w}{\partial p^3} + \frac{1}{8}\frac{\partial^3 w}{\partial q^3} + \frac{1}{27}\frac{\partial^3 w}{\partial r^3} = -\sin(p+2q+3r)\,cost - 3\cos(p+2q+3r)\,sint,$$
$$t > 0, \quad 0 < \alpha \le 1 \quad (34)$$

Along with the starting condition

$$w(p,q,r,0) = 1$$

Step 1: Apply Laplace Transform

Taking Laplace transform with respect to t,

$$L\left[\frac{\partial^{\alpha} w}{\partial t^{\alpha}}\right] = u^{\alpha} L[w(p,q,r,t)] - u^{\alpha-1} w(p,q,r,0).$$

Starting condition is

$$u^{\alpha} L[w(p,q,r,t)] - u^{\alpha-1} = L[-\sin(p+2q+3r)\,cost] - L[3\cos(p+2q+3r)\,sint]$$
$$-L\left[\frac{\partial^3 w}{\partial p^3} + \frac{1}{8}\frac{\partial^3 w}{\partial q^3} + \frac{1}{27}\frac{\partial^3 w}{\partial r^3}\right] \quad (35)$$

Hence

$$L[w(p,q,r,t)] = \frac{1}{u} + \frac{1}{u^{\alpha}} L[-\sin(p+2q+3r)\,cost] - \frac{1}{u^{\alpha}} L[3\cos(p+2q+3r)\,sint]$$
$$-\frac{1}{u^{\alpha}} L\left[\frac{\partial^3 w}{\partial p^3} + \frac{1}{8}\frac{\partial^3 w}{\partial q^3} + \frac{1}{27}\frac{\partial^3 w}{\partial r^3}\right] \quad (36)$$

Upon applying the reverse Laplace transform

$$w(p,q,r,t) = 1 + L^{-1}\left[\frac{1}{u^{\alpha}} L[-\sin(p+2q+3r)\,cost]\right] - L^{-1}\left[\frac{1}{u^{\alpha}} L[3\cos(p+2q+3r)\,sint]\right]$$
$$-L^{-1}\left[\frac{1}{u^{\alpha}} L\left[\frac{\partial^3 w}{\partial p^3} + \frac{1}{8}\frac{\partial^3 w}{\partial q^3} + \frac{1}{27}\frac{\partial^3 w}{\partial r^3}\right]\right] \quad (37)$$

Step 2: Apply Adomian Decomposition Method

Assume

$$w(p,q,r,t) = \sum_{k=0}^{\infty} w_k(p,q,r,t)$$

$$w_0(p,q,r,t) = 1 + L^{-1}\left[\frac{1}{u^{\alpha}} L[-\sin(p+2q+3r)\,cost]\right]$$

$$cost = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \cdots$$

$$w_0(p,q,r,t) = 1 - \sin(p+2q+3r)\left[\frac{t^{\alpha}}{\Gamma\alpha+1} - \frac{t^{\alpha+2}}{\Gamma\alpha+3} + \frac{t^{\alpha+4}}{\Gamma\alpha+5} - \frac{t^{\alpha+6}}{\Gamma\alpha+7} + \cdots\right] \quad (38)$$

Step 3: Compute w_1

$$w_1(p,q,r,t) = -L^{-1}\left[\frac{1}{u^{\alpha}} L[3\cos(p+2q+3r)\,sint]\right] - L^{-1}\left[\frac{1}{u^{\alpha}} L\left[\frac{\partial^3 w}{\partial p^3} + \frac{1}{8}\frac{\partial^3 w}{\partial q^3} + \frac{1}{27}\frac{\partial^3 w}{\partial r^3}\right]\right]$$

Therefore

$$\frac{\partial^3 w}{\partial p^3} + \frac{1}{8}\frac{\partial^3 w}{\partial q^3} + \frac{1}{27}\frac{\partial^3 w}{\partial r^3} = -3\cos(p+2q+3r)\,G(t)$$

$$G(t) = \frac{t^{\alpha}}{\Gamma\alpha+1} - \frac{t^{\alpha+2}}{\Gamma\alpha+3} + \cdots. \tag{39}$$

which exactly cancels the forcing term contribution. Hence,

$$w_1(p,q,r,t) = 0$$
$$w_2 = w_3 = \cdots. = 0$$

Thus final solution

$$w(p,q,r,t) = 1 - \sin(p+2q+3r)\left[\frac{t^{\alpha}}{\Gamma\alpha+1} - \frac{t^{\alpha+2}}{\Gamma\alpha+3} + \frac{t^{\alpha+4}}{\Gamma\alpha+5} - \frac{t^{\alpha+6}}{\Gamma\alpha+7} + \cdots\right]$$

Special Case: $\alpha = 1$

When,

$$w(p,q,r,t) = 1 - \sin(p+2q+3r)\, sint \tag{40}$$

Which is the exact solution in the classical sense.

Conclusion

This study derives exact closed-form solutions for cubic order dispersive non-integer order partial derivative equations using the Laplace–Adomian Decomposition Method (LADM). The nonlocal derivatives are treated in the Caputo sense. The method was successfully applied to both fractional-order and integer-order cases, and the obtained results showed excellent agreement with the exact solutions. The numerical examples demonstrated the effectiveness, reliability, and accuracy of LADM in solving fractional partial differential equations. It was further observed that, as the fractional order approaches an integer value, the fractional solutions gradually converge to the corresponding exact classical solutions. The investigation also indicated that fractional-order models often describe physical and experimental phenomena more realistically than integer-order models. Another important observation is that varying the fractional order allows the construction of more suitable mathematical models for representing experimental data and practical systems. Therefore, meaningful and realistic interpretations can be achieved through fractional modeling.

Overall, the study confirms that L-ADM is a powerful and effective method for obtaining solutions of F-PDEs compared with methods such as A-D-M, V-I-M, and D-T-M reported in the previous studies. The approach provides rapid convergence and highly accurate approximations to exact solutions. In future research, LADM may also be extended to solve other categories of nonlinear non-integer order partial differential equations encountered in scientific research, technological applications, and computational mathematics. The obtained fractional-order solutions can contribute to a deeper comprehension of many observable processes modeled by F-PDEs.

References

[1]. Ball, J.M.; Chen, G.Q.G. Entropy and convexity for nonlinear partial differential equations. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 2013.

[2]. Djidjeli, K.; Price, W.G.; Twizell, E.H.; Wang, Y. Numerical methods for the solution of the third- and fifth-order dispersive Korteweg–de Vries equations. *Journal of Computational and Applied Mathematics* 1995, 58, 307–336.

[3]. Gaxiola, O.G. The Laplace–Adomian decomposition method applied to the Kundu–Eckhaus equation. *International Journal of Mathematics and Its Applications* 2017, 5, 1–12.

[4]. Goswami, A.; Singh, J.; Kumar, D. Numerical simulation of fifth-order KdV equations occurring in magneto-acoustic waves. *Ain Shams Engineering Journal* 2018, 9, 2265–2273.

[5]. Hilfer, R. *Applications of Fractional Calculus in Physics*; World Scientific Publishing: River Edge, NJ, USA, 2000.

[6]. Jafari, H.; Khalique, C.M.; Nazari, M. Application of the Laplace decomposition method for solving linear and nonlinear fractional diffusion–wave equations. *Applied Mathematics Letters* 2011, 24, 1799–1805.

[7]. Jiang, J.; Feng, Y.; Li, S. Exact solutions to the fractional differential equations with mixed partial derivatives. *Axioms* 2018, 7, 10.

[8]. Kanth, A.R.; Aruna, K. Solution of fractional third-order dispersive partial differential equations. *Egyptian Journal of Basic and Applied Sciences* 2015, 2, 190–199.

[9]. Kocak, H.; Pinar, Z. On solutions of the fifth-order dispersive equations with porous medium type non-linearity. *Waves in Random and Complex Media* 2018, 28, 516–522.

[10]. Lopes, A.M.; Machado, J.A.T. Entropy analysis of soccer dynamics. *Entropy* 2019, 21, 187.

[11]. Machado, J.T. Entropy analysis of integer and fractional dynamical systems. *Nonlinear Dynamics* 2010, 62, 371–378.

[12]. Miller, K.S.; Ross, B. *An Introduction to the Fractional Calculus and Fractional Differential Equations*; Wiley: New York, NY, USA, 1993.

[13]. Mohamed, M.Z. Comparison between the Laplace decomposition method and Adomian decomposition in time-space fractional nonlinear fractional differential equations. *Applied Mathematics* 2018, 9, 448.

[14]. Pandey, R.K.; Mishra, H.K. Homotopy analysis Sumudu transform method for time-fractional third-order dispersive partial differential equation. *Advances in Computational Mathematics* 2017, 43, 365–383.

[15]. Prakash, A.; Kumar, M. Numerical method for fractional dispersive partial differential equations. *Communications in Numerical Analysis* 2017, 1, 1–18.

[16]. Seadawy, A.R. New exact solutions for the KdV equation with higher-order nonlinearity by using the variational method. *Computers & Mathematics with Applications* 2011, 62, 3741–3755.

[17]. Shi, Y.; Xu, B.; Guo, Y. Numerical solution of Korteweg–de Vries–Burgers equation by the compact-type CIP method. *Advances in Difference Equations* 2015, 2015, 353.

[18]. Sibatov, R.; Shulezhko, V.; Svetukhin, V. Fractional derivative phenomenology of percolative phonon-assisted hopping in two-dimensional disordered systems. *Entropy* 2017, 19, 463.

[19]. Silva, F.; Moreira, D.; Moret, M. Conformable Laplace transform of fractional differential equations. *Axioms* 2018, 7, 55.

[20]. Steudel, H.; Drazin, P.G.; Johnson, R.S. *Solitons: An Introduction*; Cambridge University Press, 1989.

[21]. Sultana, T.; Khan, A.; Khandelwal, P. A new non-polynomial spline method for solution of linear and non-linear third-order dispersive equations. *Advances in Difference Equations* 2018, 2018, 316.

[22]. Thabet, H.; Kendre, S.; Chalishajar, D. New analytical technique for solving a system of nonlinear fractional partial differential equations. *Mathematics* 2017, 5, 47.

[23]. Ubriaco, M.R. Entropies based on fractional calculus. *Physics Letters A* 2009, 373, 2516–2519.

[24]. Wazwaz, A.M. An analytic study on the third-order dispersive partial differential equations. *Applied Mathematics and Computation* 2003, 142, 511–520.

[25]. Yavuz, M.; Özdemir, N. European vanilla option pricing model of fractional order without singular kernel. *Fractal and Fractional* 2018, 2, 3.

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