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Optimization of Inventory Costs under Exponential Deterioration, Shortage, Inflation, and Power Dependent Demand using AI- Based Techniques

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Abstract

This research introduces dynamic inventory model that combines exponential linear power dependent demand, in which the demand rate depends on the selling price. The model describes the reality of market behaviour in that items that decay with time are taken into account and demand changes with the decisions made on prices. In order to reflect the current economic environment, inflation is also factored in by use of present value discounting methods to allow a better evaluation of the inventory related expenses throughout the planning time frame. Particle Swarm Optimization (PSO) algorithms based on Artificial Intelligence (AI) are further used to optimize the model to find the cost minimizing inventory policy. Shortages are also allowed and are assumed to be fully backlogged, and therefore the model is applicable when delayed fulfilment is tolerated in industries. The entire numerical example is presented to show the structure of calculations and prove the efficiency of the methodology. Besides that, an analysis of the effect of the most important parameters on the total cost of the system is carried out through a sensitivity analysis; these parameters include the rate of deterioration and inflation, demand exponent, holding cost and shortage cost. The findings indicate that the joint impact of deterioration, inflation and price-sensitive demand play a significant role in optimal ordering decisions. The optimization with the use of AI provides more powerful and efficient solutions compared to the traditional mathematical methods. In general, this research may be useful to managers and practitioners who need to address perishable or deteriorating products in inflationary conditions so that they could implement cost-efficient and evidence-based decision-making procedures.

Keywords: *Deterioration, Inflation, Power-dependent demand, EOQ, PSO Algorithm, Shortage.*
Mathematics Subject Classification (MSC) 2020: *90B05, 90C26, 68T20, 91B24.*

INTRODUCTION

In recent years, effective inventory management has received much attention for perishable or deteriorating items such as pharmaceuticals, dairy products, or radioactive materials. These items lose value over time, either due to physical decay or technological obsolescence. Classical inventory Economic order quantity (EOQ) models, often assuming constant demand, fixed costs, and negligible deterioration. However, such assumptions are not realistic for many industries.

Over time, research has evolved to accommodate various practical complexities. One such enhancement is the inclusion of exponential deterioration, where the product decay accelerates with time, as shown by (1). For a constant rate of deterioration system an order-level inventory model has been presented by (2-4). Incorporating time dependent rate of deterioration, Inventory models were developed by (5-6). Some of the noteworthy recent work in this area have been done by (7-17). Initially, classical models assumed fixed demand that often varies with time and stock. For price dependent demand, economic lot size model was developed in (18). An inventory model designed to enhance items for a price-dependent demand rate was proposed by (19). Inspired from work of (18) and (19), there was introduced An EOQ model for Weibull-deteriorating products with demand dependent on power by (20), where The demand rate is a function of the selling price but did not consider inflation. Inflation is a decisive component in long-term planning, as it erodes the purchasing power of money.

On the other hand, inflation was incorporated into a multi-echelon deteriorating inventory model in (21) but did not integrate power-dependent demand or exponential deterioration in a classical EOQ framework. Also advances in AI have become influence in inventory modelling. PSO has been successfully applied to non-linear cost functions for more accurate solutions for complex systems (22).

To address these gaps, this research proposes a new model that combines:

- Exponential deterioration (time-based decay),
- Power-dependent demand (sensitive to selling price),
- Inflationary conditions (incorporating the time value of money), and
- AI-based optimization techniques (using PSO Algorithm) to solve the nonlinear cost function.

By integrating these aspects, the paper develops a comprehensive, inflation-aware inventory model that reflects realistic supply chain behaviour and optimizes cost using modern computational techniques. The model is analytically derived and numerically validated, providing strategic insights for decision-makers handling deteriorating products in inflationary economies.

METHODOLOGY

In this study, methodology follows a systematic framework in following stages:

Problem Conceptualization and Assumptions

This stage involves identifying the real-world factors of inventory systems that deals with deterioration and inflation. Some assumptions are considered for this proposed model.

Mathematical Model Formulation

This stage involves construction of total cost function where inventory level was formulated for two phases, i.e. stock period and shortage period. Deterioration losses were derived using Weibull based deterioration functions. Also Cost components include purchasing cost, ordering cost, holding cost and shortage cost. Total cost function $TC(T, s)$ was as a nonlinear function of cycle length T, selling price s and deterioration parameters.

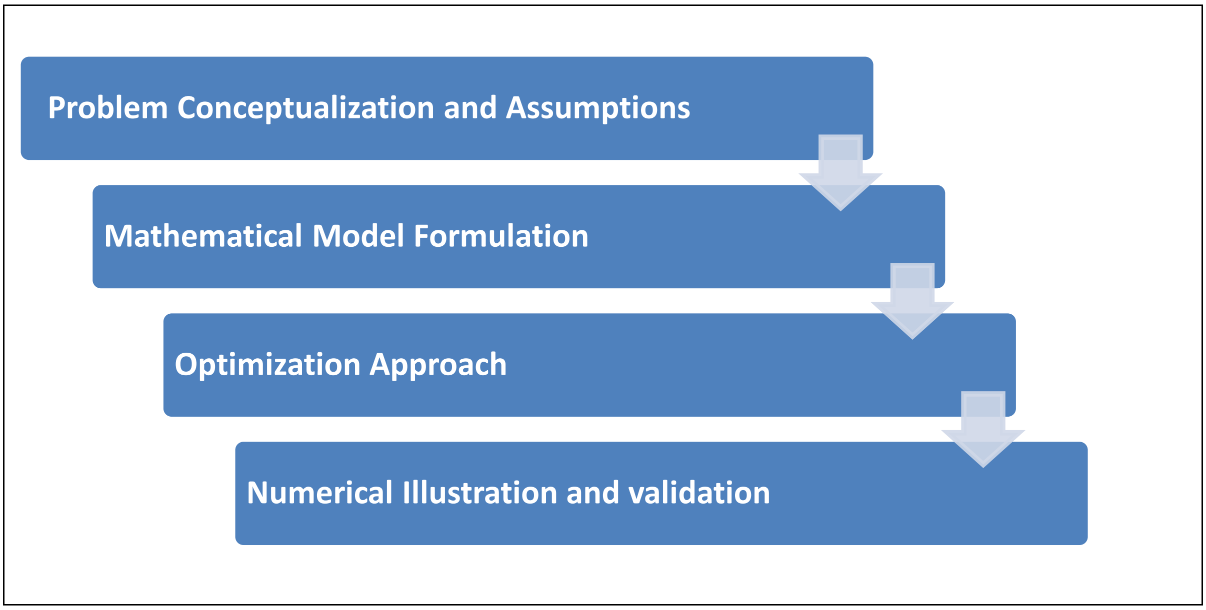
Optimization Approach

The formulated total cost function is nonlinear which is much difficult to be solved using classical calculus-based methods. Therefore, two optimization techniques were applied. First, analytical optimization was used where the first-order conditions were derived to obtain the necessary optimality conditions. These conditions are given by:

$$\frac{\partial}{\partial T} TC(T, s) = 0 \qquad \frac{\partial}{\partial s} TC(T, s) = 0$$

However, due to the nonlinear nature of the cost function, obtaining a closed-form solution becomes challenging. To overcome this limitation, a metaheuristic optimization approach was also implemented using the PSO algorithm. PSO was applied due to its efficiency in handling nonlinear and multiparameter inventory models. For computational analysis, Python programming was used to execute algorithm. A numerical example was provided using some data by help of running the PSO algorithm. Also, sensitivity analysis was conducted by varying key parameters deterioration Rate α and Inflation Rate r by 10% and 20%. Impact of parameter changes on total inventory cost was done to verify robustness.

Figure 1: Methodology Flowchart



ASSUMPTIONS AND NOTATIONS

To design the proposed inflation-adjusted EOQ model with power-dependent demand and exponential deterioration, the subsequent notations and assumptions are introduced here:

1. The demand rate is considered to vary with selling price which is
$$D(s) = (a - bs) > 0$$
2. Instantaneous replenishment is assumed, i.e., lead time is considered as zero.
3. Shortages are permitted and completely backlogged.
4. Inflation is considered, and the present worth of costs is calculated using a constant discount rate r > 0
5. Each cycle has a set duration T, and the planning horizon is finite.
6. Q represents Ordering size per cycle.
7. A represents Ordering cost.
8. C represents unit purchasing cost of the unit.
9. h represents the cost of holding of one unit per unit of time.
10. π represents costs associated with shortages per unit and time.
11. s represents Selling price per unit.
12. A two parameter Weibull distribution is employed to model the deterioration with instantaneous rate of replenishment is $\alpha\beta t^{\beta-1}, 0 < \alpha < 1, \beta > 0$, where α is scale parameter and β is shape parameter.

During interval t_1 inventory decreases due to demand and deterioration of the item. At time t_1 the inventory level reaches zero and shortages begin.

MATHEMATICAL FORMULATION OF THE MODEL

Let $I(t)$ denote the inventory level at any instant $t \in [0, T]$ where T is the cycle length and t_1 is the time when the becomes zero and shortage begins.

Inventory Level During $0 \le t \le t_1$

During this period, the inventory reduces due to both demand and deterioration. The differential formula is :

$$\frac{d}{dt}I(t) + \alpha\beta t^{\beta-1}I(t) = -(a - bs) \qquad 0 \le t \le t_1 \qquad (1)$$

$$\text{with } I(t_1) = 0$$

Solving this linear order differential equation neglecting higher powers of α , we get

$$\frac{a-bs}{e^{\alpha t^\beta}}\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left({t_1}^{\beta+1} - t^{\beta+1}\right)\right] \qquad 0 \le t \le t_1 \qquad (2)$$

Inventory Level During $t_1 \le t \le T$ (Shortage Period)

Once the inventory is exhausted, shortages accumulate according to the demand rate:

$$\frac{d}{dt}I(t) = -(a - bs) \qquad t_1 \le t \le T \qquad (3)$$

This equation's solution is found as:

$$I(t) = (a - bs)(t - t_1) \qquad t_1 \le t \le T \qquad (4)$$

Loss Due to Deterioration

The total deterioration loss over the cycle length T is:

$$L(T) = \int_0^{t_1} \delta(t)\, I(t)dt = \int_0^{t_1} \alpha\beta\, t^{\beta-1}I(t)dt$$

$$= \int_0^{t_1} \alpha\beta\, t^{\beta-1}(a - bs)(t_1 - t)\, dt$$

$$= \alpha\beta(a - bs)\int_0^{t_1} t^{\beta-1}(t_1 - t)\, dt$$

$$= \alpha\beta(a - bs)\frac{{t_1}^{\beta+1}}{(\beta+1)\beta} \qquad = \alpha\beta(a - bs)\frac{{t_1}^{\beta+1}}{(\beta+1)\beta}$$

$$L(T) = a - bs)\left[\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right] \qquad (5)$$

Ordering Quantity

The total order quantity Q in a cycle is the total of deterioration loss and demand during the cycle:

$$Q = L(T) + D(s)\, T$$

$$Q = (a - bs)\left[\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right] + (a - bs)T \qquad (6)$$

Holding Cost

The cost of holding during the time frame $[0, t_1]$, including inflation, is:

$$H = h\int_0^{t_1} I(t)e^{-rt}\, dt$$

where e^{-rt} denotes factor due to inflation.

$$H = h\int_0^{t_1} \frac{a - bs}{e^{\alpha t^\beta}}\left[(t_1 - t) + \frac{\alpha}{\beta + 1}\left({t_1}^{\beta+1} - t^{\beta+1}\right)\right]e^{-rt}\, dt$$

$$H = h(a - bs)\int_0^{t_1}\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left({t_1}^{\beta+1} - t^{\beta+1}\right)\right]e^{-rt-\alpha t^\beta}\, dt \qquad (7)$$

Shortage Cost

Shortage cost during the cycle is

$$S = \pi\int_{t_1}^{T} I(t)e^{-rt}dt$$

$$= \pi\int_{t_1}^{T} (a - bs)(t - t_1)e^{-rt}dt$$

$$S = \pi(a-bs)e^{-rt_1}\frac{[1-e^{-r(t-t_1)}\{1+r(T-t_1)\}]}{r^2}$$

Total Cost

Let $TC(T, t_1, s)$ denotes total inventory cost given by sum of purchasing, holding, ordering and shortage costs:

$$TC(T, t_1, s) = CQ + Ae^{-rT} + H + S$$

$$TC(T, t_1, s) = C(a-bs)\left[\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right] + (a-bs)T + Ae^{-rT}$$
$$+ h(a-bs)\int_0^{t_1}\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left(t_1{}^{\beta+1} - t^{\beta+1}\right)\right]e^{-rt-\alpha t^{\beta}}\,dt$$
$$+\pi(a-bs)e^{-rt_1}\frac{[1-e^{-r(t-t_1)}\{1+r(T-t_1)\}]}{r^2}$$

Let $t_1 = \gamma T, \ \ 0 < \gamma < 1$

Hence Total cost is

$$TC(T, s) = C(a-bs)\left[\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}\right] + C(a-bs)T + Ae^{-rT}$$
$$+ h(a-bs)\int_0^{\gamma T}\left[(\gamma T - t) + \frac{\alpha}{\beta+1}\left(\gamma^{\beta+1}T^{\beta+1} - t^{\beta+1}\right)\right]e^{-rt-\alpha t^{\beta}}\,dt$$
$$+\pi(a-bs)e^{-r\gamma T}\frac{[1-e^{-r(t-\gamma T)}\{1+r(T-\gamma T)\}]}{r^2} \qquad (8)$$

Minimizing total cost $TC(T, s)$ is objective of this model where necessary conditions are:

$$\frac{\partial}{\partial T}TC(T, s) = 0 \qquad \frac{\partial}{\partial s}TC(T, s) = 0$$

and

$$H = \begin{vmatrix} \frac{\partial^2}{\partial T^2}TC(T, s) & \frac{\partial^2}{\partial T\partial s}TC(T, s) \\ \frac{\partial^2}{\partial T\partial s}TC(T, s) & \frac{\partial^2}{\partial s^2}TC(T, s) \end{vmatrix} > 0$$

Numerical Example

Let $a = 5000, b = 2, c = 10, A = 5000, h = 0.5, \pi = 10, \alpha = 0.1, \beta = 0.5, r = 0.02,$

$$\gamma = 0.4$$

Using these data through Python software with AI based PSO Optimization, optimal values are calculated as:

$$T^* = 1, s^* = 30.00, Q^* = 5023.32, \ TC^* = 114{,}424.25$$

Sensitivity Analysis

A sensitivity analysis is carried to evaluate the effect of parameters variations on the optimal value of total cost. Changing of parameters by 10% and 20%, results are shown in Table 1 and impact of the rate of degradation α and Inflation Rate r on the Total Inventory Cost is shwon in Figure 2.

Table – 1

Analysis of the model's sensitivity

Parameter	**-20%**	**-10%**	**0%**	**+10%**	**+20%**
α (TC*)	114088.99	114256.62	114424.25	114591.88	114759.50
r (TC*)	114452.39	114438.31	114424.25	114410.21	114396.19

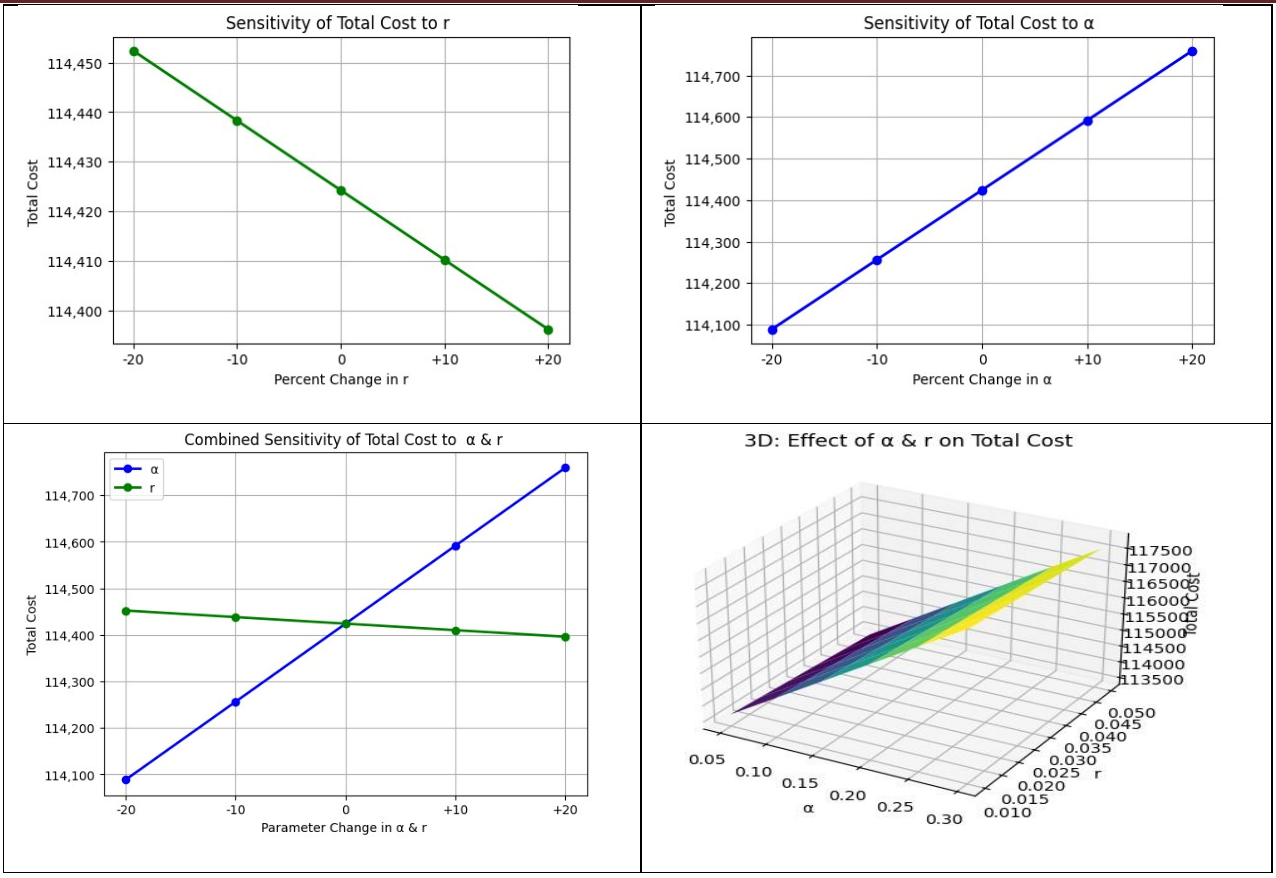


Figure 2: Graphical Representation of the Effect of Deterioration Rate α and Inflation Rate r on the Total Inventory Cost

From the sensitivity analysis, following observations are derived:

1. Increase in α leads to increase in total cost. Reason behind this is higher deterioration causes greater inventory wastage that leads increase in holding cost and subsequently in total cost.
2. Decrease in α leads to decrease in total cost. Reason behind this is Lower deterioration reduces inventory losses, enabling longer replenishment cycles and minimizing associated costs.
3. Increase in r leads to decrease in total cost, since future holding, shortage, and ordering costs are discounted more.
4. Decrease in r leads to increase in total cost, because the impact of future costs becomes heavier in present value terms.

APPLICATION

The proposed inventory model has significant applicability in real world business environments.

Industry	**Application**
Retail and E-commerce	Pricing and replenishment for perishable goods to minimize costs and wastage.
Pharmaceuticals	Managing deteriorating inventories like medicines and vaccines under inflation.
Agriculture and Food	Reducing wastage and optimizing prices for perishable agricultural products.
Manufacturing	Planning production for seasonal goods with deterioration and inflation factors.
Logistics and Warehousing	Optimizing storage and replenishment schedules under multi-location constraints.

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