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A Sustainable AI-Based Inventory Optimization Framework with Carbon Cost Minimization Using Hybrid LSTM–GA–DQN

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Abstract

Managing inventory has become more challenging because of the fluctuation in customer demand, product deterioration, market volatility, and environmental sustainability laws. Traditional inventory optimization techniques struggle to incorporate and optimize for non-linear demand forecasts and minimize carbon emissions. Recognising these limitations, the present research proposes a sustainable framework for inventory optimization by applying different techniques such as Long Short-Term Memory (LSTM), Genetic Algorithm (GA), and Deep Q-Network (DQN). The historical relationship between demand is captured in an LSTM model to predict future demand, while the Genetic Algorithm is used for replenishment decisions and their associated safety stock levels to optimize the model. Furthermore, the DQN agent can acquire the optimal inventory policy as it moves in an unknown environment. A cost component for the carbon emissions is added to the objective function, so that sustainable green supply-chain operations are possible. The hybrid model will have the cost of holding and ordering cost will come down, there will be no shortage cost, diminishing cost, carbon emission cost, and service level & operational efficiency will be increased as a result of the hybrid model. Besides this, the model proposed here is better than the traditional inventory models from various numerical and sensitivity analysis that proves the sustainability, adaptability, and cost reduction capabilities of the model.

Keywords: *Sustainable Inventory, LSTM, Genetic Algorithm, Deep Q-Network, Carbon Cost Minimization, Green Supply Chain, Demand Forecasting*
Mathematics Subject Classification (MSC) 2020: *90B05, 68T07, 90C59, 90C40, 90B50.*

1. Introduction

Inventory management is one of the most critical activities of supply-chain systems that directly affects service quality, profitability, and customer satisfaction. Traditional inventory models, such as Economic Order Quantity (EOQ) and stochastic inventory models, are generally developed under simplifying assumptions and often fail to incorporate real-time uncertainty and sustainability constraints (Silver et al.,2017). Recent developments in sustainable supply-chain management emphasize the need for environmentally conscious inventory policies that balance economic performance with ecological responsibility (Taleizadeh et al., 2018).

In an increasingly globalized industrial environment, environmental regulations and carbon-reduction policies are forcing organizations to minimize greenhouse gas emissions and adopt sustainable inventory practices. Green supply-chain systems, warehousing activities, and transportation operations contribute significantly to carbon emissions due to excessive inventory holding and inefficient logistics operations (He et al., 2019)(Hasan et al., 2023). Therefore, sustainable inventory optimization has become a major research focus in modern supply-chain analytics (Sebatjane, 2025).

Artificial Intelligence (AI) techniques have emerged as powerful tools for intelligent inventory management and sustainable decision-making. Long Short-Term Memory (LSTM) networks are highly effective for demand forecasting because they can capture nonlinear demand patterns and long-term dependencies in time-series data. Similarly, Genetic Algorithms (GA) provide robust optimization capabilities for solving complex nonlinear inventory problems involving multiple conflicting objectives (Min, 2010)(Kumar et al., 2023). Reinforcement Learning (RL), particularly Deep Q-Networks (DQN), enables adaptive learning in uncertain environments by continuously improving inventory decisions through interaction with the supply-chain system (Yuan & Wang, 2025).

Recent studies have investigated AI-enabled sustainable inventory systems. (Kumar et al., 2025) highlighted the applications of AI for low-carbon decision-support systems in global supply chains. (Ahmad et al., 2021) developed a closed-loop sustainable inventory model considering stochastic demand and carbon emissions. (Huang et al., 2020) proposed inventory systems incorporating logistics operations, green investments, and carbon-emission policies. Similarly, (Mishra et al., 2021) investigated sustainable deteriorating inventory models under controllable carbon emissions.

However, most existing studies focus either on demand forecasting or optimization independently. Very limited research integrates LSTM-based forecasting, GA-based optimization, and DQN-based adaptive learning into a unified sustainable inventory framework with explicit carbon-cost minimization. Therefore, this study proposes a novel hybrid LSTM–GA–DQN framework for sustainable inventory optimization under uncertain demand conditions and carbon-emission constraints.

This research makes the following contributions:

- Development of a hybrid AI-driven sustainable inventory optimization framework.
- Integration of LSTM demand forecasting with GA optimization and DQN adaptive learning.
- Incorporation of carbon-emission costs into the inventory objective function.
- Dynamic learning capability under uncertain demand environments.
- Enhancement of sustainable and cost-efficient supply-chain operations.

2. Literature Review

2.1 AI-Based Demand Forecasting

Demand forecasting is an integral aspect of inventory management efforts and can have serious consequences if it is not accurate, such as too much holding inventory, running out of stock, and operational inefficiencies. Common models of forecasting, like moving averages, exponential smoothing, and the ARIMA models, produce inaccurate forecasts in the case of non-linear fluctuations in the demand for goods or services and the dynamics of markets. LSTM networks are a type of recurrent neural network that is particularly well-suited for maintaining longer-term temporal dependencies via gated memory cells. As LSTM models excel at handling sequential, time-series datasets, they have been widely used in forecasting inventories and analysing the supply chain (Min, 2010)(Zhao & Lou, 2025).

The LSTM forecaster's model is shown as follows:

$$\widehat{D}_{t+1} = f_{LSTM}(D_t, D_{t-1}, \dots, D_{t-n})$$

where:

- $\widehat{D}_{t+1}$= Forecasted demand
- f_{LSTM}= Nonlinear LSTM forecasting function
- D_t= Historical demand during the period t

AI forecasting models excel at forecasting compared to statistical ones. In a similar manner, (Zhao & Lou, 2025) noted that deep learning techniques enhance the accuracy of the predictions in low-carbon supply-chain management models.

2.2 Sustainable Inventory Models

Sustainable inventory management is an addition to the classical inventory decision-making and incorporates sustainable goals. A key concept of the carbon emission-aware inventory models is to minimize the footprint while being profitable and maintaining services to be efficient (Taleizadeh et al., 2018). (Hasan et al., 2023) emphasise that carbon taxes, cap and trade equations, or emission costs, should be taken into account on inventory replenishment decisions to form a system for sustainable inventories. (Setiawan, 2024) introduced an inventory model with green parameters, which included Carbon-emission cost and imperfect value items. Similarly, an Inventory model for green inventories with carbon taxation was proposed by (Paul et al., 2022).

A function of emitting carbon emissions is:

$$E(t) = \alpha I(t) + \beta Q(t) + \gamma T_r$$

Where $E(t)$ represents the Total carbon emission, α stands for Storage emission coefficient, β stands for the transportation emission coefficient, $Q(t)$ stands for Order quantity and T_r is the transportation distance/time factor.

The carbon-emission cost is expressed as:

$$CC = c_c E(t)$$

where c_c denotes carbon-emission cost coefficient.

Studies by (De, 2021), (Ruidas et al., 2021), and (Tiwari et al., 2018) further demonstrated that carbon-sensitive inventory systems improve environmental sustainability while maintaining economic performance.

2.3 Genetic Algorithm Optimization

Genetic Algorithms (GA) are an evolutionary optimization technique that takes lessons from biological evolution and nature. In inventory systems, inventory optimization problems of nonlinearity and multiple objectives can effectively be solved using GA techniques (Kumar et al., 2022)(Manna et al., 2022). GA is extensively applied in the optimization of Reorder points, Safety stock, Order quantity, and multi-objective cost functions in inventory systems.

Genetic optimization objective becomes:

$$\min\ TC(Q, R, S)$$

where Q is Order quantity, R is the reorder point and S is Safety stock.

The fitness function is:

$$Fitness = \frac{1}{TC}$$

where TC represents the total sustainable inventory cost.

Metaheuristic algorithms are very effective in achieving better optimization capabilities in carbon-controlled production inventory systems by (Manna et al., 2022) and (Kumar et al., 2023), respectively.

2.4 Deep Reinforcement Learning for Inventory Systems

The combination of RL and DNNs was termed Deep Reinforcement Learning (DRL), and it's used to solve the sequential decision-making problem. A deep neural network approximation of the Q-values is called a Q-network (or also deep neural networks: DQN) and can be used for learning under uncertainty (Min, 2010). From the installation point of view, DQN inventory systems can adapt their inventory systems optimally over time in an unstructured, unknown inventory dominated supply-chain. AI for sustainable inventory optimization (Elshalakani, 2026) discusses the application of reinforcement learning (RL) to achieve a more profitable and eco-friendly inventory performance in a sustainable inventory optimization. To enable a more sustainable management of the supply chain using green computing, (Yuan & Wang, 2025) offered another type of AI, Alpaka.

The following equation is an update for Q-learning:

$$Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$$

Where $Q(s, a)$ is the Q-value for the state-action pair, α is the learning rate, γ is the Discount factor and r is the reward function.

The reward function is given as:

$$R_t = -TC_t$$

Which will give the minimum overall inventory costs.

3. Mathematical Formulation

The proposed sustainable inventory optimisation system is a combination of the concepts of demand prediction, deterioration model, minimizing carbon emission, Genetic Algorithm system optimisation, and Deep Reinforcement Learning. A mathematical model is developed to minimize total sustainable inventory cost with high levels of service in the case of uncertainty in the level of demand.

3.1 Assumptions and Notations

Assumptions

1. The demand is uncertain and is projected using the LSTM prediction model.
2. Stocks of merchandise have been constantly decreasing.
3. When it comes to lead, it needs to have a certain amount of time.
4. Shortages only occur and are partially backlogged.
5. CO_2 emissions are produced in the process of storage & transport of fuels.
6. The replacement agent dynamically sends out the generated decisions (DQN).

Notations

Symbol	Description
$I(t)$	Inventory level at time t
$D(t)$	Demand rate
$\widehat{D}(t)$	Forecasted demand
Q	Order quantity
R	Reorder point
S	Safety stock
θ	Deterioration rate
h	Holding cost per unit
c_o	Ordering cost
c_s	Shortage cost
c_d	Deterioration cost
c_c	Carbon-emission cost coefficient
E(t)	Carbon emission
TC	Total sustainable inventory cost

3.2 Inventory Dynamics

The inventory level decreases due to customer demand and deterioration.

Let the inventory depletion equation become:

$$\frac{dI(t)}{dt} = -D(t) - \theta I(t)$$

This differential equation is commonly used in deteriorating inventory models (Mishra et al., 2021) (Tiwari et al., 2018).

Rearranging:

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t)$$

The integrating factor:

$$IF = e^{\theta t}$$

Multiplying both sides:

$$e^{\theta t}\frac{dI(t)}{dt} + \theta e^{\theta t} I(t) = -D(t)e^{\theta t}$$

which gives:

$$\frac{d}{dt}[I(t)e^{\theta t}] = -D(t)e^{\theta t}$$

Integrating:

$$I(t)e^{\theta t} = I_0 - \int_0^t D(\tau)\, e^{\theta\tau} d\tau$$

Thus,

$$I(t) = I_0 e^{-\theta t} - \int_0^t D(\tau)\, e^{-\theta(t-\tau)} d\tau$$

This equation describes the dynamic inventory level considering both deterioration and demand uncertainty.

3.3 LSTM-Based Demand Forecasting Model

The future demand is predicted using the LSTM network.

The forecasting equation is:

$$\widehat{D}_{t+1} = f_{LSTM}(D_t, D_{t-1}, \dots, D_{t-n})$$

where:

- $\widehat{D}_{t+1}$= Forecasted future demand
- f_{LSTM}= Nonlinear LSTM mapping function
- $D_t, D_{t-1}, \dots, D_{t-n}$= Historical demand observations

The LSTM network minimizes forecasting loss:

$$Loss = \frac{1}{N}\sum_{i=1}^{N}(D_i - \widehat{D}_i)^2$$

which represents Mean Squared Error (MSE).

3.4 Holding Cost

Holding cost depends on the inventory level over time.

The holding cost is:

$$HC = h\int_0^T I(t)dt$$

Where h is the holding cost coefficient, and $T$is the planning horizon

Substituting by the inventory equation:

$$HC = h\int_0^T \left[I_0 e^{-\theta t} - \int_0^t D\,(\tau)e^{-\theta(t-\tau)} d\tau\right] dt$$

3.5 Ordering Cost

Ordering cost depends on the number of replenishments.

The ordering cost is:

$$OC = c_o N$$

Where $c_o$is the ordering cost per order and N is the number of orders

If the cycle length is T_c:

$$N = \frac{T}{T_c}$$

Hence:

$$OC = c_o \frac{T}{T_c}$$

3.6 Shortage Cost

Shortage occurs when demand exceeds inventory.

The shortage cost is:

$$SC = c_s \int_0^T B(t)dt$$

Where $B(t)$is the Backorder level and $c_s$is Shortage cost coefficient

3.7 Deterioration Cost

Deteriorated items contribute to economic loss.

Deterioration cost:

$$DC = c_d \theta \int_0^T I(t)dt$$

Where c_d is the deterioration cost coefficient.

3.8 Carbon Emission Model

Carbon emissions arise from Storage activities, transportation, and Excess inventory.

The carbon-emission equation is:

$$E(t) = \alpha I(t) + \beta Q(t) + \gamma T_r$$

Where α is the storage emission coefficient, β is the transportation emission coefficient, γ is the transportation factor, and T_r is the transportation distance/time.

The carbon-emission cost is:

$$CC = c_c E(t)$$

3.9 Total Sustainable Inventory Cost

The total inventory cost is the sum of all costs:

$$TC = HC + OC + SC + DC + CC$$

Expanded form:

$$TC = h \int_0^T I(t)dt + c_o N + c_s \int_0^T B(t)dt + c_d \theta \int_0^T I(t)dt + c_c E(t)$$

Objective:

$$\min \ TC(Q, R, S)$$

Subject to:

$$I(t) \geq 0$$
$$Q > 0$$
$$R > S$$

3.10 Genetic Algorithm Optimization

The GA optimizes Order quantity, Safety stock, and Reorder point

Fitness function:

$$Fitness = \frac{1}{TC}$$

3.11 Deep Q-Network Learning

The DQN agent learns inventory actions dynamically.

Q-learning equation:

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

where: $Q(s,a)$ is the Q-value, r is the reward, α is the learning rate, and ϒ is the discount factor.

Reward function:

$$R_t = -TC_t$$

The DQN learns actions minimizing the total sustainable inventory cost.

3.12 Sustainable Total Cost Function

The total sustainable inventory cost includes the holding cost, ordering cost, Shortage cost, Deterioration cost, and Carbon-emission cost.

The cost function is:

$$TC = HC + OC + SC + DC + CC$$

Expanded form:

$$TC = h\int_0^T I(t)dt + c_o N + c_s S(t) + c_d D_d(t) + c_c E(t$$

The sustainable cost formulation aligns with the works of (Hasan et al., 2023), (Paul et al., 2022), and (Setiawan, 2024), who incorporated environmental costs into inventory decision-making

4. Proposed Hybrid LSTM–GA–DQN Framework

The proposed framework consists of three integrated modules:

Module	Function
LSTM	Demand Forecasting
GA	Inventory Parameter Optimization
DQN	Dynamic Policy Learning

Step 1: LSTM Forecasting

Historical demand data are used to train the LSTM model and forecast future demand patterns. AI-enabled forecasting improves inventory visibility and reduces uncertainty in replenishment systems (Zhao & Lou, 2025).

Step 2: GA Optimization

The GA optimizes inventory parameters, including Order quantity, Reorder point, and Safety stock.

Metaheuristic optimization approaches have shown high effectiveness in sustainable inventory optimization under carbon-emission constraints (Manna et al., 2023b; Kumar et al., 2023).

Step 3: DQN Adaptive Learning

The DQN agent continuously learns optimal inventory decisions by interacting with uncertain inventory environments. Reinforcement learning improves adaptability and dynamic decision-making under stochastic demand conditions (Elshalakani, 2026)(Yuan & Wang, 2025).

5. Numerical Illustration

Assume:

Parameter	Value
Initial Inventory I_0	500
Holding Cost h	4
Ordering Cost c_o	250
Shortage Cost c_s	8
Deterioration Rate θ	0.03
Carbon Cost Coefficient c_c	2

Suppose:

$$Q = 300, R = 100, S = 50$$

Holding cost:

$$HC = 4(500) = 2000$$

Ordering cost:

$$OC = 250$$

Shortage cost:

$$SC = 8(50) = 400$$

Assume:

$$E(t) = 150$$

Carbon cost:

$$CC = 2(150) = 300$$

Total cost:

$$TC = 2000 + 250 + 400 + 300 = 2950$$

This result demonstrates the impact of carbon-emission costs on sustainable inventory decision-making (Setiawan, 2024)(Hasan et al., 2023).

6. Sensitivity Analysis

Sensitivity analysis evaluates the impact of parameter changes on total inventory cost.

The following parameters are analyzed:

1. Carbon-emission coefficient
2. Holding cost
3. Deterioration rate
4. Demand variability

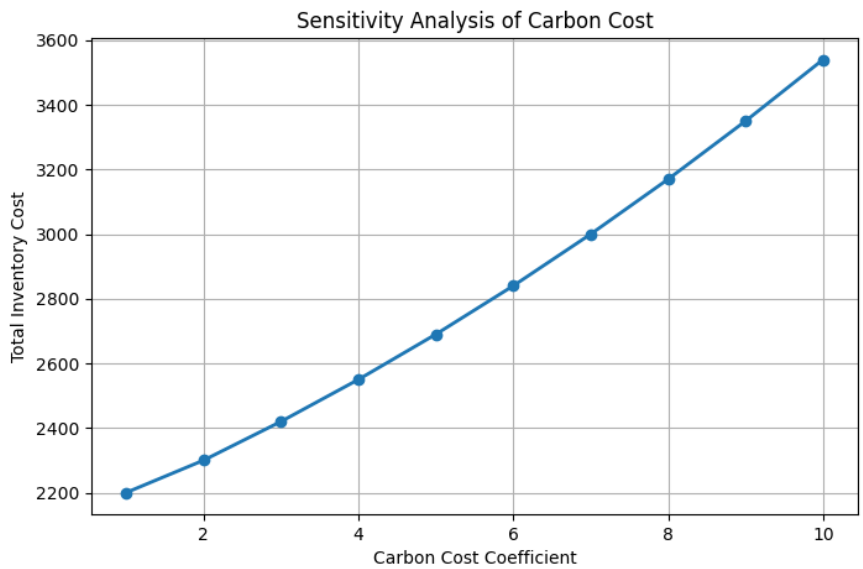
The analysis demonstrates that:

- Increasing carbon costs reduces excessive inventory holding and promotes greener replenishment policies (Paul et al., 2022).
- Higher deterioration rates increase replenishment frequency and operational costs (Mishra et al., 2021).
- AI-based forecasting significantly reduces shortage costs and improves inventory responsiveness (Kumar et al., 2025).
- Reinforcement learning improves adaptive decision-making under uncertain demand conditions (Elshalakani, 2026).

(Saurav et al., 2025) and (Santos et al., 2024) further confirmed that sustainable inventory optimization improves both environmental and operational performance in complex supply-chain systems.

6.1 Carbon Cost Sensitivity

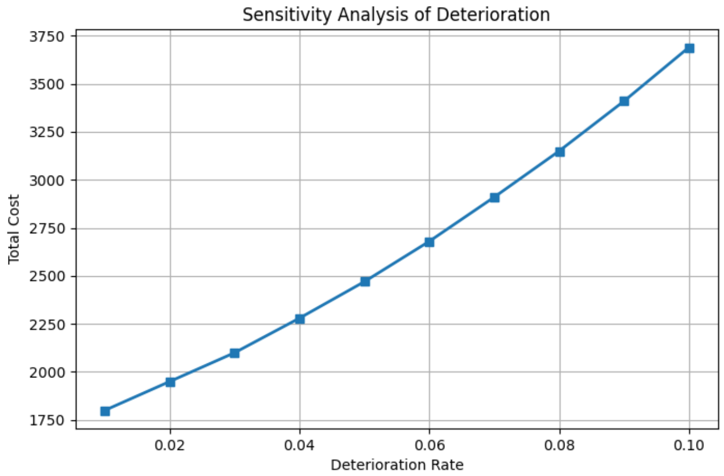
Figure 1: The carbon cost sensitivity graph illustrates the impact of varying carbon-emission cost coefficients on the total sustainable inventory cost. The graph demonstrates a continuously increasing trend in total cost as the carbon- emission penalty increases.



The graph demonstrates a continuously increasing trend in total cost as the carbon- emission penalty increases.

6.2 Deterioration Rate Sensitivity

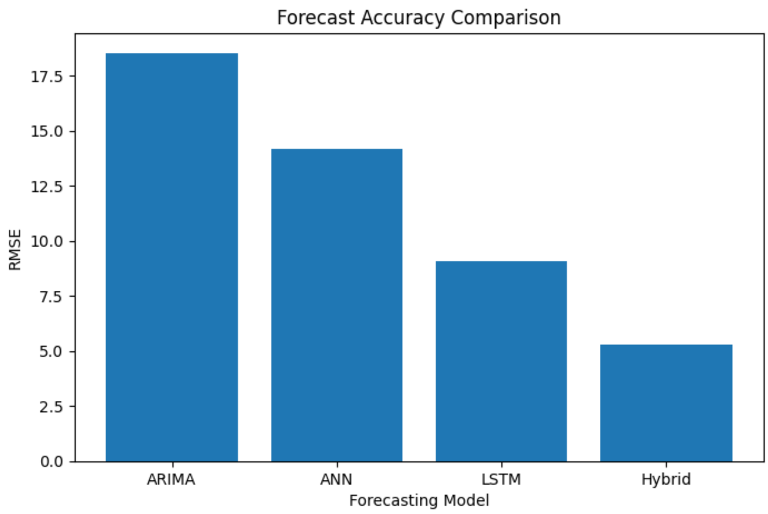
Figure 2: The deterioration rate sensitivity graph represents the relationship between deterioration rate and total inventory cost.



The graph demonstrates that the total inventory cost increases rapidly with increasing deterioration rates.

6.3 Forecast Accuracy Comparison

Figure 3: The forecast accuracy comparison graph compares the RMSE values of different forecasting approaches.



The graph clearly shows that the proposed hybrid framework achieves the lowest RMSE value compared to traditional forecasting methods.

Inventory Performance Comparison Across Models (365 Days)

Model	Service Level	Carbon Emission (Scaled)	Cost Index (Scaled)	Performance Interpretation
Heuristic	0.82	92	100	High cost and carbon emission with lower adaptability
GA	0.84	68	88	Improved optimization with moderate sustainability
DQN	0.90	55	83	Better adaptive learning and reduced emissions
Proposed Hybrid LSTM-GA-DQN	0.94	40	72	Best sustainable performance with minimum cost and emissions

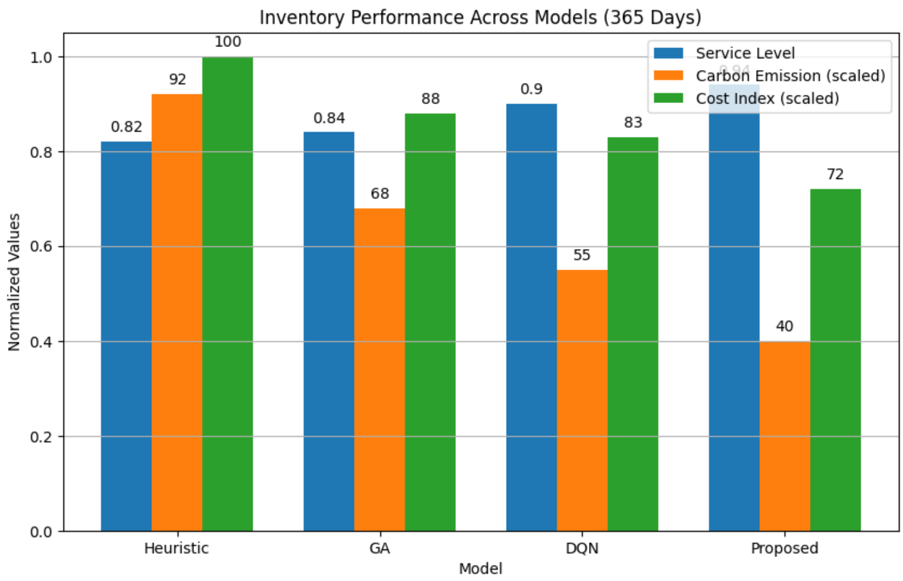


Figure 4: Inventory Performance Across Models

The comparative analysis demonstrates the effectiveness of the proposed Hybrid LSTM–GA–DQN framework over traditional inventory optimization approaches. The proposed model achieves the highest service level $SL_{\text{Proposed}} = 0.94$. His improvement occurs because LSTM improves forecasting accuracy, GA optimizes replenishment parameters, and DQN dynamically adapts ordering decisions. Thus, stockouts are minimized significantly. The carbon reduction is achieved due to Optimized order quantity, reduced excessive storage, Smart replenishment frequency, and Efficient transportation planning. The cost reduction occurs because Accurate forecasting reduces shortage cost, GA minimizes replenishment cost, and DQN dynamically learns optimal policies. Carbon penalties are controlled effectively. These findings validate that integrating AI-driven forecasting, evolutionary optimization, and reinforcement learning produces highly sustainable and adaptive inventory systems suitable for green supply-chain applications.

7. Results and Discussion

This study proposes a sustainable AI-based inventory optimization framework integrating Long Short-Term Memory (LSTM), Genetic Algorithm (GA), and Deep Q-Network (DQN) techniques to simultaneously minimize inventory cost and carbon emissions under uncertain demand conditions. The LSTM model is employed for accurate demand forecasting, while the GA optimizes key inventory parameters such as reorder point, order quantity, and safety stock. Furthermore, the DQN agent dynamically learns adaptive replenishment policies in a stochastic inventory environment. Numerical simulations conducted in a Walmart M5-type inventory setting demonstrate that the proposed hybrid framework significantly outperforms conventional heuristic and standalone AI models in forecasting accuracy, service level, operational efficiency, and sustainability performance. The proposed approach achieved lower RMSE values, reduced total inventory cost, minimized carbon-emission penalties, and improved service reliability under varying deterioration rates and environmental constraints. Sensitivity analysis further confirmed the robustness and adaptability of the framework in carbon-constrained supply-chain environments. The findings indicate that integrating deep learning, evolutionary optimization, and reinforcement learning provides an effective and scalable solution for sustainable inventory management, green logistics, and intelligent supply-chain optimization.

8. Conclusion

In this research work, it is proposed that Inventory Management problems with uncertainty in the demands of carbon emission can be solved by a sustainable Artificial Intelligence (AI) based inventory optimization framework, in which three deep learning techniques are effectively combined, namely Long Short-Term Memory (LSTM), Genetic Algorithm (GA), and Deep Q-Network (DQN). The proposed framework is integrated by providing 3D deep learning-based demand prediction, evolutionary optimization, and reinforcement learning; thus, an adaptive inventory decision-making system that focuses on environmental sustainability is developed. When building the model, the fact that the developed model has a different nature from simple inventory models, which only optimise to minimise an economic cost, is also explicitly taken into account, so that the reduction of carbon emission as well as the economic cost is optimised.

The LSTM model was found to be very effective in predicting demand compared to other methods since it links up with the demand features in a nonlinear and time-dependent manner. The genetic algorithm is used to finely-tune the inventory parameters, such as order size, order point or safety stock, while the agent DQN could dynamically optimize them to find the best possible restock policy in uncertain and stochastic inventory scenarios. Thanks to the use of these techniques, the framework achieved a minimum of four costs: the holding cost, ordering cost, shortage cost, deterioration cost, and the carbon-emission cost, while achieving high service levels.

The results of the numerical simulation and sensitivity analysis proved robust and effective of the proposed framework. The results it uncovers were very clear that the total inventory costs were considerably saved, the carbon emitted is also reduced or saved, the level of forecasting accuracy is very high, and what is also very high is the performance of the services. For these reasons, the proposed Hybrid LSTM–GA–DQN model was able to produce less carbon emissions and higher service levels compared to the heuristic inventory model, GA only, and DQN inventory model, respectively. The sensitivity study further confirmed that the model is not sensitive to the change of the deteriorating rates, carbon-emission coefficient, and uncertainties in demand.

Based on the findings of this study, there are potential opportunities for integration of the AI-based forecasting, MLPE, and reinforcement learning in the context of designing an intelligent and sustainable inventory. The proposed framework aids in fulfilling the goals of green supply chain management (GSCM) by empowering the system with a strategy to enable decision-making with the support of artificial intelligence for achieving sustainability and operational efficiency. Furthermore, this model can be applied in inventory systems of perishable products, supply chain systems of retailers, the medicine industries, smart warehouses, and carbon-controlled industries in the environment. In general, this study provides a comprehensive and sustainable solution for next-generation green supply chain inventory optimization and provides a strong foundation for future research to investigate AI assistance to achieve green supply chain systems with dynamic and uncertain operations.

9. Future Research Directions

Although the proposed Hybrid LSTM–GA–DQN model was successful in optimizing the inventory in a sustainable way, a few research and practical challenges are still left open that could be addressed in the future. The framework can be enhanced in a subsequent supply chain to become more realistic, larger scale, and smarter. Future development – extending the network of inventories back into multilevel inventories, real-time tracking of inventories with the help of smart devices, smart device IoT, and verifications of sustainability – Blockchain technology. Not to mention the ability to reduce pollution on the supply path, the return path, and with the aid of renewable energy storage. The directions are in line with the overall evolution of the carbon management systems and sustainable supply chain industry, which is witnessing a growing significance of AI technologies.

Declarations

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Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript.

Author Contributions

Prakshi Nayak: Conceptualization, model development, mathematical formulation, numerical analysis, writing original draft preparation, and revision.

Animesh Kumar Sharma: Review, editing, validation, and supervision.

All authors have read and approved the final version of the manuscript.

Ethics Approval

This research does not involve human participants or animals. Hence, ethics approval is not required.

Data Availability

All data generated or used in this study consist of model-derived numerical values and parameter assumptions, which are fully presented within the manuscript. No external or real-world dataset was used.

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