Pi International Journal of Mathematical Sciences
Volume 2 | Issue 3 | June 2026 | ISSN: 3107-9830 (Online)
Available online at: https://pijms.com/
DOI: https://doi.org/10.59828/pijms.v2i3.38

Soft $\mathcal{G}$ Hausdorff, Soft $\mathcal{G}$ Normal and Soft $\mathcal{G}$ Regular Via Soft Grill Topological Spaces

N. Chandramathi[1], **P. Nithya**[2] **and V. Kiruthika**[3]

[1]Assistant Professor, Department of mathematics, Government Arts College, Udumalpet, Tamilnadu, India.
Email: drmathimaths@gmail.com
[2]Research scholar, Department of mathematics, Government Arts College, Udumalpet, Tamilnadu, India.
Email: nithyapalanisamy2599@gmail.com
[3]Research scholar, Department of mathematics, Government Arts College, Udumalpet, Tamilnadu, India.
Email: kiruthi.v3@gmail.com
(**Corresponding author**: P. Nithya[2])

Received: 17 May 2026 | **Accepted:** 18 June 2026 | **Published:** 30 June 2026

Abstract

In this paper, we introduce and study the concept of soft $\zeta_S - \mathcal{G}$ Hausdorff, soft $\zeta_S - \mathcal{G}$ regular and soft $\zeta_S - \mathcal{G}$ normal in soft topological spaces with soft grill. We establish several fundamental properties and examine the implications among these classes of spaces.

Keywords: *Soft sets, soft $\zeta_S - \mathcal{G}$ closed set, $\zeta_S - \mathcal{G}$ Hausdorff, soft $\zeta_S - \mathcal{G}$ regular and soft $\zeta_S - \mathcal{G}$ normal.*
Mathematics Subject Classification (MSC) 2020: *54A05, 54A10, 54D10.*

I. Introduction

Soft topological spaces were first studied by Shabir et-al [8] in 2011. soft open and soft closed sets, soft subspace, soft closure, soft neighborhood of a point, soft τ_i-spaces, for i = 1, 2, 3, 4, soft regular spaces, soft normal spaces were among the fundamental concept of soft topological spaces that they created and established. In soft grill topological spaces, Rodyna A.Hosny[9] has introduced by soft regular and soft Hausdorff spaces. In this study, we present a new notion in soft grill topological spaces named $\zeta_S - \mathcal{G}$ normal, $\zeta_S - \mathcal{G}$ Regular and $\zeta_S - \mathcal{G}$ Hausdorff along with several related with theorems and examples.

II Preliminaries

Definition 2.1 [8]

A non empty collection $\zeta \subseteq SS(X, A)$ of soft sets over X is called a soft grill, if the following conditions hold:

(i) If $F_A \in \zeta$ and $F_A \subseteq H_A$, which implies $H_A \in \zeta$.

(ii) If $F_A \subseteq H_A \in \zeta$, which implies $F_A \in \zeta$ or $H_A \in \zeta$.

Definition 2.2 [3]

Let $\zeta_{\mathcal{S}}$ be a soft grill over a soft topological space (X, $\tau_{\mathcal{S}}$, $\mathcal{A}$). A soft set $\mathcal{F}_{ß}$ is called $\zeta_{\mathcal{S}}$ generalized closed set (briefly $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set), if $\chi_{\zeta}(\mathcal{F}_{ß}) \subseteq \mathcal{U}_{\mathcal{A}}$, whenever $\mathcal{F}_{ß} \subseteq \mathcal{U}_{\mathcal{A}}$ and $\mathcal{U}_{\mathcal{A}}$ is soft open in $(\mathcal{X}, \tau_{\mathcal{S}}, \mathcal{A})$. The complement of such set will be called $\zeta_{\mathcal{S}} - \mathcal{G}$ open set (resp. $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open set).

Definition 2.3 [4]

A function δ_{pu}: $(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}$, A)→ $(Y, \sigma_{\mathcal{S}}$, B) is called soft generalized continuous functions in soft grill topological spaces (shortly $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous) if for soft closed set L_B of $(Y, \sigma_{\mathcal{S}}$, B), $\delta_{pu}^{-1}((L_B) \in \zeta_{\mathcal{S}} - \mathcal{G}C$(X, $\tau_{\mathcal{S}}, \zeta_{\mathcal{S}}$, A).

Definition 2.4 [4]

A function δ_{pu}: $(\mathcal{X}, \tau_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^1, \mathcal{A})$→ (Y, $\sigma_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^2, \mathcal{B}$) is called soft generalized irresolute in soft grill topological spaces (shortly $\zeta_{\mathcal{S}} - \mathcal{G}$ irresolute) if for soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set $L_{\mathcal{B}}$ of (Y, $\sigma_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^2, \mathcal{B}$), $\delta_{pu}^{-1}((L_{\mathcal{B}}) \in \zeta_{\mathcal{S}} - \mathcal{G}C\left(\mathcal{X}, \tau_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^1, \mathcal{A}\right)$.

Definition 2.5[10]

Let (X,τ, A) be a soft topological spaces over X, let (G,A) be a soft closed set in X and soft point E_α^x such that $E_\alpha^x \notin$ (G, A). If there exist a two soft open sets (F_1, A) and (F_2, A) such that $E_\alpha^x \in (F_1, A)$, (G,A)⊆ (F_2, A) and $(F_1, A) \cap (F_2, A)$=∅, then (X,τ, A) is called soft regular space.

Definition 2.6 [10]

Let (X,τ, A) be a soft topological spaces over X, let (G_1,A) and (G_2,A)be a two disjoint soft closed sets. If there exist a two soft open sets (F_1, A) and (F_2, A) such that (G_1,A)⊆ (F_1, A), (G_2,A)⊆ (F_2, A) and $(F_1, A) \cap (F_2, A)$=∅, then (X,τ, A) is called soft normal space.

Definition 2.7[9]

Let $\mathcal{G}$ be a soft grill on a soft topological spaces over (X,τ, E). Then, the space (X,τ, E) is called soft $\mathcal{G}$ regular if for any soft closed set F in $\tilde{X}$ with soft point $x \notin$ F, there exist disjoint soft open sets U and V such that $x \in$ U and (F-V)$\notin \mathcal{G}$.

Definition 2.8 [1]

Let (X,τ, E) be a soft topological space and $x, y \in$ X such that $x \neq y$. (X,τ, E) is called soft Hausdorff spaces or soft T_2-space if there exist a soft τ open sets F and G such that $x \in$ F, $y \in$ G and F∩G=∅.

III. $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff Space

In this section, we define a novel category of soft generalized Hausdorff space within the framework of soft grills as follows.

Definition 3.1

Let $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ be a soft grill topological space and $x_{\mathcal{A}}, y_{\mathcal{A}} \in \mathcal{X}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \neq y_{\mathcal{A}}$. If $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is called soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff spaces or soft $\zeta_{\mathcal{S}} - \mathcal{G} - T_2$-space if there exist a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{F}_{\mathcal{A}}, y_{\mathcal{A}} \in \mathcal{H}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} \cap \mathcal{H}_{\mathcal{A}}=\emptyset_{\mathcal{A}}$.

Example 3.2

Let $\mathcal{X} =\{ \mu_1, \mu_2, \mu_3 \}and\mathcal{A}=\{ \alpha_1\}$, $\tau_{\mathcal{S}} = \{\emptyset_{\mathcal{A}}, \mathcal{X}_{\mathcal{A}}, \{\mu_1\}, \{\mu_2\}, \{\mu_3\}, \{\mu_1, \mu_2\}, \{\mu_2, \mu_3\}, \{\mu_1, \mu_3\}\}$, and $\zeta_{\mathcal{S}}=\{\{\mu_1, \mu_2\}, \{\mu_2, \mu_3\}, \mathcal{X}_{\mathcal{A}}\}$. Now $x_{\mathcal{A}} = \mu_1$, $y_{\mathcal{A}} = \mu_2$, $\mathcal{F}_{\mathcal{A}} = \{\mu_1\}$, $\mathcal{H}_{\mathcal{A}} = \{\mu_2, \mu_3\}$ and $\mathcal{F}_{\mathcal{A}}, \mathcal{H}_{\mathcal{A}}$ are soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets such that $x_{\mathcal{A}} \in \mathcal{F}_{\mathcal{A}}, y_{\mathcal{A}} \in \mathcal{H}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} \cap \mathcal{H}_{\mathcal{A}}=\emptyset_{\mathcal{A}}$. Therefore $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff spaces.

Theorem 3.3

Every soft Hausdorff space is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space.

Proof

Let $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ be a soft grill topological space. Since every soft open set is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set, it follows that $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space.

The converse of the above theorem is not true as seen from the following example.

Example 3.4

Let $\mathcal{X}=\{\mu_1, \mu_2, \mu_3\}$, $\mathcal{A}=\{\alpha_1\}$, $\tau_{\mathcal{S}} = \{\emptyset_{\mathcal{A}}, \mathcal{X}_{\mathcal{A}}, \{\mu_1\}, \{\mu_2, \mu_3\}, \{\mu_1, \mu_3\}\}$, and $\zeta_{\mathcal{S}}=\{\{\mu_1, \mu_2\}, \{\mu_2, \mu_3\}, \mathcal{X}_{\mathcal{A}}\}$. Then $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space but not soft Hausdorff space.

Theorem 3.5

Every soft subspace of soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space.

Proof

Assume that $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ be a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space and $\mathcal{Y}_{\mathcal{A}}$ be a soft subspace of $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$. Since $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space, r_1 and r_2 are the two soft point of $\mathcal{Y}_{\mathcal{A}}$. By hypothesis, there exist soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets $\mathcal{E}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}}$ in $\mathcal{X}_{\mathcal{A}}$, $r_1 \in \mathcal{E}_{\mathcal{A}}$, $r_2 \in \mathcal{F}_{\mathcal{A}}$, $\mathcal{E}_{\mathcal{A}} \cap \mathcal{F}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$. So, $\mathcal{E}_{\mathcal{A}} \cap \mathcal{Y}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} \cap \mathcal{Y}_{\mathcal{A}}$ are soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets in $\mathcal{Y}_{\mathcal{A}}$. So $r_1 \in \mathcal{E}_{\mathcal{A}}, r_2 \in \mathcal{F}_{\mathcal{A}}$, gives $r_1 \in \mathcal{E}_{\mathcal{A}} \cap \mathcal{F}_{\mathcal{A}}$. Similarly $r_2 \in \mathcal{F}_{\mathcal{A}}, r_2 \in \mathcal{Y}_{\mathcal{A}}$ gives $r_2 \in \mathcal{F}_{\mathcal{A}} \cap \mathcal{Y}_{\mathcal{A}}$. But $\mathcal{E}_{\mathcal{A}} \cap \mathcal{F}_{\mathcal{A}}$ is a empty set, $(\mathcal{Y}_{\mathcal{A}} \cap \mathcal{F}_{\mathcal{A}}) \cap (\mathcal{Y}_{\mathcal{A}} \cap \mathcal{E}_{\mathcal{A}})= \mathcal{Y}_{\mathcal{A}} \cap (\mathcal{E}_{\mathcal{A}} \cap \mathcal{F}_{\mathcal{A}}) = \mathcal{Y}_{\mathcal{A}} \cap \emptyset_{\mathcal{A}} = \emptyset_{\mathcal{A}}$. Hence $\mathcal{E}_{\mathcal{A}} \cap \mathcal{Y}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} \cap \mathcal{Y}_{\mathcal{A}}$ are disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets in $\mathcal{Y}_{\mathcal{A}}$ such that $r_1 \in \mathcal{E}_{\mathcal{A}} \cap \mathcal{Y}_{\mathcal{A}}$ and $r_2 \in \mathcal{F}_{\mathcal{A}} \cap \mathcal{Y}_{\mathcal{A}}$. Hence $\mathcal{Y}_{\mathcal{A}}$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space.

Theorem 3.6

Let $\mathcal{X}_{\mathcal{A}}$ be a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ Hausdorff space. Consider the two soft discrete points r_1 and r_2 of $\mathcal{X}_{\mathcal{A}}$, then there is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{F}_{\mathcal{A}}$ where r_1 is a element of $\mathcal{F}_{\mathcal{A}}$ and $y_{\mathcal{A}} \notin \zeta_{\mathcal{S}} - \mathcal{G} - cl\ (\mathcal{F}_{\mathcal{A}})$.

Proof

Consider any two soft discrete points r_1 and r_2 of $\mathcal{X}_{\mathcal{A}}$, where $r_1 \neq r_2$. But $\mathcal{X}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$ Hausdorff space, then there exist two soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{E}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}$ in $\mathcal{X}_{\mathcal{A}}$ where r_1 is an element of $\mathcal{E}_{\mathcal{A}}$ and r_2 is an element of $\mathcal{F}_{\mathcal{A}}$. Hence $\mathcal{F}_{\mathcal{A}}{}^C$ is a soft $\zeta_S - \mathcal{G}$ closed set, gives $\zeta_S - \mathcal{G} - cl\ (\mathcal{F}_{\mathcal{A}}) \subseteq \mathcal{F}_{\mathcal{A}}{}^C$. Hence r_2 is an element of $\mathcal{F}_{\mathcal{A}}{}^C$. Thus r_2 is not a element of $\zeta_S - \mathcal{G} - cl\ (\mathcal{F}_{\mathcal{A}})$.

IV. $\zeta_S - \mathcal{G}$ REGULAR SPACE

In this section, we define a novel category of soft generalized regular space within the framework of soft grills as follows.

Definition 4.1

Let ζ_S be a soft grill on a topological spaces $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$. A space $\mathcal{X}_{\mathcal{A}}$ is said to be a soft $\zeta_S - \mathcal{G}$ regular, if for each pair consisting of a point $x_{\mathcal{A}}$ and a soft $\zeta_S - \mathcal{G}$ closed set $\mathcal{F}_{\mathcal{A}}$ not containing $x_{\mathcal{A}}$, there exist disjoint soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}$and $\mathcal{F}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}} \notin \zeta_S$.

Example 4.2

Let $\mathcal{X}$={μ$_1$, μ$_2$}, $\mathcal{A}$ ={α$_1$, α$_2$}, $\tau_S = \{\emptyset_{\mathcal{A}}, \mathcal{X}_{\mathcal{A}}, K_1, K_2, K_3, K_4\}$ and $\zeta_S=\{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \mathcal{X}_{\mathcal{A}}\}$, where $K_1, K_2, K_3, K_4, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6$ are soft subsets over $\mathcal{X}_{\mathcal{A}}$, we get the following $K_1 = \{\{\mu_1\}, \{\mu_2\}\}$, $K_2=\{\{\mu_2\}, \{\mu_1\}\}$, $K_3 = \{\emptyset, \{\mu_1\}\}$, $K_4 = \{\{\mu_1\}, \mathcal{X}\}$, $\rho_1 = \{\{\mu_1\}, \{\mu_1\}\}$, $\rho_2 = \{\{\mu_2\}, \{\mu_1, \mu_2\}\}$, $\rho_3 = \{\{\mu_2\}, \{\mu_2\}\}$, $\rho_4 = K_2$, $\rho_5 = K_1$, $\rho_6 = K_4$, then $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$ is soft $\zeta_S - \mathcal{G}$ regular.

Remark 4.3

Every soft regular spaces is soft $\zeta_S - \mathcal{G}$ regular, but the converse may not be true.

Example 4.4

Let $\mathcal{X}=\{\mu_1, \mu_2, \mu_3\}$ and $\mathcal{A}=\{\alpha_1, \alpha_2\}$, $\tau_S = \{\emptyset_{\mathcal{A}}, \mathcal{X}_{\mathcal{A}}, K_1, K_1, K_2, K_3, K_4, K_5, K_6\}$, and $\zeta_S=\{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \mathcal{X}_{\mathcal{A}}\}$, where $K_1, K_2, K_3, K_4, K_5, K_6, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8$ are soft subsets over $\mathcal{X}_{\mathcal{A}}$, we get the following: $K_1 = \{\{\mu_1\}, \{\mu_1\}\}$, $K_2=\{\{\mu_2\}, \{\mu_2\}\}$, $K_3 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$, $K_4 = \{\{\mu_2, \mu_3\}, \{\mu_2, \mu_3\}\}$, $K_5 = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_3\}\}$, $K_6 = \{\{\mu_3\}, \{\mu_3\}\}$, $\rho_1 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_3\}\}$, $\rho_2 = \{\{\mu_1\}, \{\mu_1\}\}$, $\rho_3 = \{\{\mu_2\}, \{\mu_2\}\}$, $\rho_4 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$, $\rho_5 = \{\{\mu_1\}, \{\mu_2, \mu_3\}\}$, $\rho_6 = \{\{\mu_1\}, \{\mu_1, \mu_3\}\}$, $\rho_7 = \{\{\mu_1, \mu_2\}, \{\mu_1\}\}$, $\rho_8 = \{\mathcal{X}, \{\mu_1, \mu_3\}\}$. Then $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$ is soft $\zeta_S - \mathcal{G}$ regular, but not soft regular spaces

Remark 4.5

Every soft ζ_S regular spaces is soft $\zeta_S - \mathcal{G}$ regular, but the converse may not be true.

Example 4.6

Let $\mathcal{X}$={μ_1, μ_2, μ_3}, $\mathcal{A}$ ={α_1, α_2},$\tau_{\mathcal{S}}$ ={$\emptyset_{\mathcal{A}}, \mathcal{X}_{\mathcal{A}}, K_1, K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}$} and $\zeta_{\mathcal{S}}$={$\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \mathcal{X}_{\mathcal{A}}$}, where $K_1, K_2, K_3, K_4, K_5, K_6,$ $K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8$ are soft subsets over $\mathcal{X}_{\mathcal{A}}$,we get the following: $K_1 = \{\{\mu_1\}, \{\mu_1\}\}$, K_2=$\{\{\mu_2\}, \{\mu_2\}\}$, $K_3 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$, $K_4 = \{\{\mu_2, \mu_3\}, \{\mu_2, \mu_3\}\}$, $K_5 = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_3\}\}$, $K_6 = \{\{\mu_3\}, \{\mu_3\}\}$, $K_7 = \{\{\mu_1\}, \{\mu_2\}\}$, $K_8 = \{\{\mu_1, \mu_2\}, \{\mu_2\}\}$, $K_9 = \{\{\mu_1\}, \{\mu_3\}\}$, $K_{10} = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_3\}\}$,$K_{11} = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_2\}\}$, $K_{12} = \{\{\mu_2\}, \{\mu_1, \mu_3\}\}$, $\rho_1 = \{\{\mu_1\}, \{\mu_1, \mu_2\}\}$, $\rho_2 = \{\{\mu_1\}, \{\mu_1\}\}$, $\rho_3 = \{\{\mu_2\}, \{\mu_2\}\}$, $\rho_4 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$, $\rho_5 = \{\{\mu_1\}, \{\mu_2, \mu_3\}\}$, $\rho_6 = \{\{\mu_1\}, \{\mu_1, \mu_3\}\}$, $\rho_7 = \{\{\mu_1, \mu_2\}, \{\mu_1\}\}$, $\rho_8 = \{ \mathcal{X}, \{\mu_1, \mu_3\}\}$. Then $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ regular, but not soft $\zeta_{\mathcal{S}}$ regular spaces

Theorem 4.7

Let $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ be a soft grill topological space and $x_{\mathcal{A}} \in \mathcal{X}_{\mathcal{A}}$. Then the following are equivalent:

1) $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ regular.
2) For every soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set $\mathcal{F}_{\mathcal{A}}$ not containing $x_{\mathcal{A}}$, there exist disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$.

Proof

(1)→(2): Let $\mathcal{F}_{\mathcal{A}}$ be a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set such that $x_{\mathcal{A}} \cap \mathcal{F}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$. Then $x_{\mathcal{A}} \notin \mathcal{F}_{\mathcal{A}}$. By hypothesis, there exist disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$.

(2)→(1): Let $\mathcal{F}_{\mathcal{A}}$ be a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set such that $x_{\mathcal{A}} \notin \mathcal{F}_{\mathcal{A}}$. Then $x_{\mathcal{A}} \cap \mathcal{F}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$. From (2), there exist soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$. Therefore, $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ regular.

Theorem 4.8

Let $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ be a soft grill topological space and $x_{\mathcal{A}} \in \mathcal{X}_{\mathcal{A}}$. Then the following are equivalent:

1) $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ regular.
2) For every soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{F}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{F}_{\mathcal{A}}$, there exist a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{U}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}$ and $\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{U}_{\mathcal{A}}) - \mathcal{F}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$.
3) For every soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set $\mathcal{M}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \notin \mathcal{M}_{\mathcal{A}}$, there exist a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \notin \mathcal{U}_{\mathcal{A}}$ and $\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{U}_{\mathcal{A}}) \cap \mathcal{M}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$.

Proof

(1)→(2): Let $\mathcal{F}_{\mathcal{A}}$ be a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set such that $x_{\mathcal{A}} \in \mathcal{F}_{\mathcal{A}}$. Then ${\mathcal{F}_{\mathcal{A}}}^{C}$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set such that $x_{\mathcal{A}} \notin {\mathcal{F}_{\mathcal{A}}}^{C}$. It follows by (1), there exist disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}$ and

${\mathcal{F}_{\mathcal{A}}}^{C} - \mathcal{V}_{\mathcal{A}} = {\mathcal{V}_{\mathcal{A}}}^{C} - \mathcal{F}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$. Since $\mathcal{U}_{\mathcal{A}} \cap \mathcal{V}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$, then, $\mathcal{U}_{\mathcal{A}} \subseteq {\mathcal{V}_{\mathcal{A}}}^{C}$. So $\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{U}_{\mathcal{A}}) \subseteq {\mathcal{V}_{\mathcal{A}}}^{C}$ Hence $\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{U}_{\mathcal{A}}) - \mathcal{F}_{\mathcal{A}} \subseteq {\mathcal{V}_{\mathcal{A}}}^{C} - \mathcal{F}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$.

(2)→(3): Let $\mathcal{M}_{\mathcal{A}}$ be a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set such that $x_{\mathcal{A}} \notin \mathcal{M}_{\mathcal{A}}$. Then ${\mathcal{M}_{\mathcal{A}}}^{C}$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set such that $x_{\mathcal{A}} \in {\mathcal{M}_{\mathcal{A}}}^{C}$. From(2), there exist a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{U}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}$ and $\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{U}_{\mathcal{A}}) - {\mathcal{M}_{\mathcal{A}}}^{C} = \zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{U}_{\mathcal{A}}) \cap \mathcal{M}_{\mathcal{A}} \notin \zeta_{\mathcal{S}}$.

(3)→(1): Let $\mathcal{V}_{\mathcal{A}}$ be a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set such that $x_{\mathcal{A}} \notin \mathcal{V}_{\mathcal{A}}$. From (3), there exist a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{F}_{\mathcal{A}}$ such that $x_{\mathcal{A}} \in \mathcal{F}_{\mathcal{A}}$ and $\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{F}_{\mathcal{A}}) \cap \mathcal{V}_{\mathcal{A}} = \mathcal{V}_{\mathcal{A}} \ -(\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{F}_{\mathcal{A}}))^{C} \notin \zeta_{\mathcal{S}}$, where $\mathcal{F}_{\mathcal{A}}$ and $(\zeta_{\mathcal{S}} - \mathcal{G} - cl(\mathcal{F}_{\mathcal{A}}))^{C}$ are disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets. Therefore, $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ regular.

V. $\zeta_{\mathcal{S}} - \mathcal{G}$ NORMAL SPACE

In this section, we define a novel category of soft generalized normal space within the framework of soft grills as follows

Definition 5.1

Let $\zeta_{\mathcal{S}}$ be a soft grill on a topological spaces $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$. A space $\mathcal{X}$ is said to be a soft generalized normal space in soft grill topological space(shortly soft $\zeta_{\mathcal{S}} - \mathcal{G}$ normal), if for every pair of disjoint $\zeta_{\mathcal{S}} - \mathcal{G}$ closed sets $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{J}_{\mathcal{A}}$, there exist a disjoint $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ such that $\mathcal{I}_{\mathcal{A}} \subseteq \mathcal{U}_{\mathcal{A}}$ and $\mathcal{J}_{\mathcal{A}} \subseteq \mathcal{V}_{\mathcal{A}}$.

Example 5.2

Let $\mathcal{X}$=$\{\mu_1, \mu_2, \mu_3\}$ and $\mathcal{A}$ =$\{\alpha_1, \alpha_2\}$, $\tau_{\mathcal{S}} = \{\emptyset_A, X_A, , K_1, K_2, K_3, K_4, K_5, K_6, K_7\}$, and $\zeta_{\mathcal{S}}$=$\{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8, \widetilde{\mathcal{X}}\}$, where $K_1, K_2, K_3, K_4, K_5, K_6, K_7, \rho_1, \rho_2, \rho_3, \rho_4$, $\rho_5, \rho_6, \rho_7, \rho_8$ are soft subsets over $X_{\mathcal{A}}$, we get the following: $K_1 = \{\{\mu_1\}, \{\mu_1\}\}$, K_2=$\{\{\mu_2\}, \{\mu_2\}\}$, $K_3 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$, $K_4 = \{\{\mu_2, \mu_3\}, \{\mu_2, \mu_3\}\}$, $K_5 = \{\{\mu_1, \mu_3\}, \{\mu_1, \mu_3\}\}$, $K_6 = \{\{\mu_3\}, \{\mu_3\}\}$, $\rho_1 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_3\}\}$, $\rho_2 = \{\{\mu_1\}, \{\mu_1\}\}$, $\rho_3 = \{\{\mu_2\}, \{\mu_2\}\}$, $\rho_4 = \{\{\mu_1, \mu_2\}, \{\mu_1, \mu_2\}\}$, $\rho_5 = \{\{\mu_1\}, \{\mu_2, \mu_3\}\}$, $\rho_6 = \{\{\mu_1\}, \{\mu_1, \mu_3\}\}$, $\rho_7 = \{\{\mu_1, \mu_2\}, \{\mu_1\}\}$, $\rho_8 = \{\mathcal{X}, \{\mu_1, \mu_3\}\}$, then K_1 and K_2 is disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set, K_2 and K_5 is a disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets such that $K_1 \subseteq K_5$, $K_2 \subseteq K_2$.

Hence $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ normal space.

Theorem 5.3

For a soft grill topological space $(\mathcal{X}, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, \mathcal{A})$ the following are equivalent:

1) $\mathcal{X}_{\mathcal{A}}$ is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ normal space.
2) For each pair of disjoint $\zeta_{\mathcal{S}} - \mathcal{G}$ closed sets $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{J}_{\mathcal{A}}$, there exist a disjoint soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ such that $\mathcal{I}_{\mathcal{A}} \subseteq \mathcal{U}_{\mathcal{A}}$ and $\mathcal{J}_{\mathcal{A}} \subseteq \mathcal{V}_{\mathcal{A}}$.
3) For each soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set $\mathcal{I}_{\mathcal{A}}$ and any soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open sets V_A composing $\mathcal{U}_{\mathcal{A}}$, a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set $\mathcal{U}_{\mathcal{A}}$ becomes available so that $\mathcal{I}_{\mathcal{A}} \subseteq \mathcal{U}_{\mathcal{A}} \subseteq {\tau_{\zeta}}^{\mathcal{S}} - cl(\mathcal{U}_{\mathcal{A}}) \subseteq \mathcal{V}_{\mathcal{A}}$.

Proof

(1)→(2): For each soft open set is soft $\zeta_S - \mathcal{G}$ open set.

(2)→(3): Assume that $\mathcal{I}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$ closed set and $\mathcal{V}_{\mathcal{A}}$ is a soft open set in $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$ such that $\mathcal{I}_{\mathcal{A}} \subseteq \mathcal{V}_{\mathcal{A}}$. Then $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{X}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}}$ are disjoint soft $\zeta_S - \mathcal{G}$ closed sets and so by (2), there exist disjoint soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{W}_{\mathcal{A}}$ such that $\mathcal{I}_{\mathcal{A}} \subseteq \mathcal{U}_{\mathcal{A}}$and $\mathcal{X}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}} \subseteq \mathcal{W}_{\mathcal{A}}$. Since $\mathcal{W}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$ open set and $\mathcal{X}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}} \subseteq \mathcal{W}_{\mathcal{A}}$, where $\mathcal{X}_{\mathcal{A}} - \mathcal{V}_{\mathcal{A}}$ is soft $\zeta_S - \mathcal{G}$ closed set, we have by [1] theorem, $\mathcal{X}_{\mathcal{A}} - V_A \subseteq {\tau_\zeta}^S$-int$(\mathcal{W}_{\mathcal{A}})$, so $\mathcal{X}_{\mathcal{A}} - {\tau_\zeta}^S$-int$(\mathcal{W}_{\mathcal{A}}) \subseteq \mathcal{V}_{\mathcal{A}}$. Again, $\mathcal{U}_{\mathcal{A}} \cap \mathcal{W}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$, which implies that $\mathcal{U}_{\mathcal{A}} \cap {\tau_\zeta}^S$-int$(\mathcal{W}_{\mathcal{A}}) = \emptyset_{\mathcal{A}}$, so ${\tau_\zeta}^S - cl(\mathcal{U}_{\mathcal{A}}) \subseteq \mathcal{X}_{\mathcal{A}} - {\tau_\zeta}^S$-int$(\mathcal{W}_{\mathcal{A}}) \subseteq \mathcal{V}_{\mathcal{A}}$. Thus $I_A \subseteq \mathcal{U}_{\mathcal{A}} \subseteq {\tau_\zeta}^S - cl(\mathcal{U}_{\mathcal{A}}) \subseteq \mathcal{V}_{\mathcal{A}}$, where $\mathcal{U}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$open set.

(3)→(1): Let $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{J}_{\mathcal{A}}$ are any two disjoint soft $\zeta_S - \mathcal{G}$ closed set in $\mathcal{X}_{\mathcal{A}}$. Then by (3), there exist a soft $\zeta_S - \mathcal{G}$ open set $\mathcal{U}_{\mathcal{A}}$ such that $\mathcal{I}_{\mathcal{A}} \subseteq \mathcal{U}_{\mathcal{A}} \subseteq {\tau_\zeta}^S - cl(\mathcal{U}_{\mathcal{A}}) \subseteq \mathcal{X}_{\mathcal{A}} - \mathcal{J}_{\mathcal{A}}$. Therefore $\mathcal{U}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$ open set and $\mathcal{I}_{\mathcal{A}} \subseteq \mathcal{U}_{\mathcal{A}}$ where $\mathcal{I}_{\mathcal{A}}$ is a soft closed set in $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$, we have by [1] theorem that $\mathcal{I}_{\mathcal{A}} \subseteq {\tau_\zeta}^S$-int$(\mathcal{U}_{\mathcal{A}})$. Again, SS$(\mathcal{X}_{\mathcal{A}})- \emptyset_{\mathcal{A}} \subseteq \zeta_S$ implies that ${\tau_\zeta}^S \subseteq \tau^\alpha = {\tau_\zeta}^Sint(\mathcal{U}_{\mathcal{A}})$, $\mathcal{X}_{\mathcal{A}} - {\tau_\zeta}^S - cl(\mathcal{U}_{\mathcal{A}}) \in \tau^\alpha$. Thus $\mathcal{I}_{\mathcal{A}} \subseteq {\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}}) \subseteq$ int(cl(int(${\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}}$))))= $\mathcal{M}_{\mathcal{A}}$ and $\mathcal{J}_{\mathcal{A}} \subseteq \mathcal{X}_{\mathcal{A}} - ({\tau_\zeta}^S - cl(\mathcal{U}_{\mathcal{A}})) \subseteq$ int(cl(int(${\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}}$))))= $\mathcal{H}_{\mathcal{A}}$. To show that $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$ is normal space, it now suffices to show that $\mathcal{M}_{\mathcal{A}} \cap \mathcal{H}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$. In fact $x_{\mathcal{A}} \in \mathcal{M}_{\mathcal{A}} \cap \mathcal{H}_{\mathcal{A}}$ implies that $x_{\mathcal{A}} \in$ int(cl(int(${\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}}$)))) and $x_{\mathcal{A}} \in \mathcal{H}_{\mathcal{A}}$ implies that $x_{\mathcal{A}} \in$cl(int(${\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}})$)) and $x_{\mathcal{A}} \in \mathcal{H}_{\mathcal{A}} \in \tau_S$, there exist $y_{\mathcal{A}} \in$ int(${\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}})$) and $y_{\mathcal{A}} \in \mathcal{H}_{\mathcal{A}} \subseteq$ cl(int($\mathcal{X}_{\mathcal{A}} - ({\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}}$)))) implies that int(${\tau_\zeta}^S -$int $(\mathcal{U}_{\mathcal{A}})$)$\cap$ $\mathcal{X}_{\mathcal{A}} - ({\tau_\zeta}^S - int(\mathcal{U}_{\mathcal{A}})$))$\neq \emptyset_{\mathcal{A}}$, ${\tau_\zeta}^S -$int $(\mathcal{U}_{\mathcal{A}}) \cap \mathcal{X}_{\mathcal{A}} - \left({\tau_\zeta}^S - cl(\mathcal{U}_{\mathcal{A}})\right) \neq \emptyset_{\mathcal{A}}$. Which is a contradictions.

Theorem 5.4

If $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$ is a soft $\zeta_S - \mathcal{G}$ normal space and $\mathcal{Y}_{\mathcal{A}} \in SS(\mathcal{X}_{\mathcal{A}})$ is a soft closed then $(\mathcal{Y},\ \tau_y,\ \zeta_S,\ \mathcal{A})$ is a soft $\zeta_S - \mathcal{G}$ normal space.

Proof

Let $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}}$ be a disjoint soft $\zeta_S - \mathcal{G}$ closed on $\mathcal{X}_{\mathcal{A}}$. Since $\mathcal{Y}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$ closed, $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}}$ be a disjoint soft $\zeta_S - \mathcal{G}$ closed on $\mathcal{X}_{\mathcal{A}}$. By hypothesis there exist soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{J}_A$ such that $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{I}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}} \subseteq \mathcal{J}_A$. Then $\mathcal{F}_{\mathcal{A}} - \mathcal{I}_{\mathcal{A}} = \mathcal{K}_{\mathcal{A}} \notin \zeta_S$ and$\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{I}_{\mathcal{A}} \cup \mathcal{K}_{\mathcal{A}}$. Therefore, $\mathcal{F}_{\mathcal{A}} - (\mathcal{Y}_{\mathcal{A}} \cap \mathcal{I}_{\mathcal{A}}) \subseteq (\mathcal{Y}_{\mathcal{A}} \cap \mathcal{K}_{\mathcal{A}}) \notin \zeta_S$. Similarly, $\mathcal{H}_{\mathcal{A}} - (\mathcal{Y}_{\mathcal{A}} \cap \mathcal{J}_A) \notin \zeta_S$. Hence $(\mathcal{Y},\ \tau_y,\ \zeta_S,\ \mathcal{A})$ is a soft $\zeta_S - \mathcal{G}$ normal space.

Theorem 5.5

If $(\mathcal{X}, \tau_S, \zeta_S, \mathcal{A})$ is an soft grill topological. If $\mathcal{F}_{\mathcal{A}}$ is a soft closed and $\mathcal{H}_{\mathcal{A}}$ be a soft $\zeta_S - \mathcal{G}$ closed set such that $\mathcal{F}_{\mathcal{A}} \cap \mathcal{H}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$, then there exist disjoint soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{J}_A$ such that $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{I}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}} \subseteq \mathcal{J}_A$.

Proof

Since $\mathcal{F}_{\mathcal{A}} \cap \mathcal{H}_{\mathcal{A}} = \emptyset_A$, $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{X}_{\mathcal{A}} - \mathcal{H}_{\mathcal{A}}$ where $\mathcal{X}_{\mathcal{A}} - \mathcal{H}_{\mathcal{A}}$ is soft $\zeta_S - \mathcal{G}$ open. By hypothesis, $cl_S(\mathcal{F}_{\mathcal{A}}) \subseteq X_A - \mathcal{H}_{\mathcal{A}}$. Since $cl_S\,(\mathcal{F}_{\mathcal{A}}) \cap \mathcal{H}_{\mathcal{A}} = \emptyset_{\mathcal{A}}$ and $\mathcal{X}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$ normal , there exist disjoint soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{I}_{\mathcal{A}}$ and $\mathcal{J}_A$such that $cl_S\,(\mathcal{F}_{\mathcal{A}}) \subseteq \mathcal{I}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}} \subseteq \mathcal{J}_A$. Thus $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{I}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}} \subseteq \mathcal{J}_A$.

Theorem 5.6

Let $\delta_{pu}: (\mathcal{X}, \tau_S, {\zeta_S}^1, \mathcal{A}) \rightarrow (\mathcal{Y}, \sigma_S,\ {\zeta_S}^2, \mathcal{B})$ be a soft $\zeta_S - \mathcal{G}$ homeomorphism function. If $(\mathcal{X}, \tau_S, {\zeta_S}^1, \mathcal{A})$ is soft $\zeta_S - \mathcal{G}$ normal space, then $(\mathcal{Y}, \sigma_S,\ {\zeta_S}^2, \mathcal{B})$ is image of $\mathcal{X}_{\mathcal{A}}$ is soft $\zeta_S - \mathcal{G}$ normal space.

Proof

Let two disjoint soft subsets $\mathcal{I}_{\mathcal{B}}$ and $\mathcal{J}_{\mathcal{B}}$ in $\mathcal{Y}_{\mathcal{B}}$. Given δ_{pu} is a soft $\zeta_S - \mathcal{G}$ homeomorphism, let $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{A}}$ is a soft $\zeta_S - \mathcal{G}$ closed sets in $\mathcal{X}_{\mathcal{A}}$ defined as $\mathcal{H}_{\mathcal{A}} = {\delta_{pu}}^{-1}(\mathcal{J}_{\mathcal{B}})$ such that, $\mathcal{F}_{\mathcal{A}} \cap \mathcal{H}_{\mathcal{A}} = {\delta_{pu}}^{-1}(\mathcal{I}_{\mathcal{B}} \cap \mathcal{J}_{\mathcal{B}})$= ${\delta_{pu}}^{-1}(\emptyset_{\mathcal{A}}) = \emptyset_{\mathcal{A}}$. By hypothesis, there exist soft open sets $\mathcal{K}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ in X_A such that $\mathcal{F}_{\mathcal{A}} - \mathcal{K}_{\mathcal{A}} \notin {\zeta_S}^1$ and $\mathcal{H}_{\mathcal{A}} - \mathcal{L}_{\mathcal{A}} \notin {\zeta_S}^1$. Then $\delta_{pu}(\mathcal{F}_{\mathcal{A}} - \mathcal{K}_{\mathcal{A}}) = \delta_{pu}(F_A) - \delta_{pu}(\mathcal{K}_{\mathcal{A}})\ \notin \zeta_S$ and $\delta_{pu}(\mathcal{H}_{\mathcal{A}} - \mathcal{L}_{\mathcal{A}}) = \delta_{pu}(\mathcal{H}_{\mathcal{A}}) - \delta_{pu}(\mathcal{L}_{\mathcal{A}})\ \notin \zeta_S$. Since δ_{pu} is a soft $\zeta_S - \mathcal{G}$ homeomorphism. Then $\delta_{pu}(\mathcal{K}_{\mathcal{A}}) = \mathcal{V}_{\mathcal{B}}$ and $\delta_{pu}(\mathcal{L}_{\mathcal{A}}) = \mathcal{W}_{\mathcal{B}}$ are disjoint soft $\zeta_S - \mathcal{G}$ open sets in $\mathcal{Y}_{\mathcal{B}}$, then $(\mathcal{Y}, \sigma_S,\ {\zeta_S}^2, \mathcal{B})$ is image of $\mathcal{X}_{\mathcal{A}}$ is soft $\zeta_S - \mathcal{G}$ normal space.

Theorem 5.7

If $\delta_{pu}: (\mathcal{X}, \tau_S, {\zeta_S}^1, \mathcal{A}) \rightarrow (\mathcal{Y}, \sigma_S,\ {\zeta_S}^2, \mathcal{B})$ be a soft $\zeta_S - \mathcal{G}$ irresolute . If $(\mathcal{X}, \tau_S, {\zeta_S}^1, \mathcal{A})$ is soft $\zeta_S - \mathcal{G}$ normal space, then $(\mathcal{Y}, \sigma_S,\ {\zeta_S}^2, \mathcal{B})$ is soft $\zeta_S - \mathcal{G}$ normal space.

Proof

Let $\mathcal{K}_{\mathcal{B}}$ and $\mathcal{H}_{\mathcal{B}}$ be any disjoint soft $\zeta_S - \mathcal{G}$ closed sets of $\mathcal{Y}_{\mathcal{B}}$. Then ${\delta_{pu}}^{-1}(\mathcal{K}_{\mathcal{B}})$ and ${\delta_{pu}}^{-1}(\mathcal{H}_{\mathcal{B}})$ are disjoint soft $\zeta_S - \mathcal{G}$ closed sets of $\mathcal{Y}_{\mathcal{B}}$,since δ_{pu} is soft $\zeta_S - \mathcal{G}$ irresolute function. Since $\mathcal{X}_{\mathcal{A}}$ is a $\zeta_S - \mathcal{G}$ normal space, there exist a disjoint disjoint soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{U}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}$ in $\mathcal{X}_{\mathcal{A}}$ such that ${\delta_{pu}}^{-1}(\mathcal{K}_{\mathcal{B}}) \subseteq \mathcal{U}_{\mathcal{A}}$ and ${\delta_{pu}}^{-1}(\mathcal{H}_{\mathcal{B}}) \subseteq \mathcal{V}_{\mathcal{A}}$. By[2] theorem, there exist soft $\zeta_S - \mathcal{G}$ open sets $\mathcal{I}_{\mathcal{B}}$ and $\mathcal{J}_{\mathcal{B}}$ of $\mathcal{Y}_{\mathcal{B}}$ such that $\mathcal{K}_{\mathcal{B}} \subseteq \mathcal{I}_{\mathcal{B}}$, $\mathcal{H}_{\mathcal{B}} \subseteq \mathcal{J}_{\mathcal{B}}$, ${\delta_{pu}}^{-1}(\mathcal{I}_{\mathcal{B}}) \subseteq \mathcal{U}_{\mathcal{A}}$, ${\delta_{pu}}^{-1}(\mathcal{J}_{\mathcal{B}}) \subseteq \mathcal{V}_{\mathcal{A}}$. Then we have ${\delta_{pu}}^{-1}(\mathcal{I}_{\mathcal{B}}) \cap\ {\delta_{pu}}^{-1}(\mathcal{J}_{\mathcal{B}}) = \emptyset_{\mathcal{A}}$ and hence $\mathcal{I}_{\mathcal{B}} \cap \mathcal{J}_{\mathcal{B}} = \emptyset_{\mathcal{A}}$. Therefore $\mathcal{Y}_{\mathcal{A}}$ is soft $\zeta_S - \mathcal{G}$ normal space.

VI. CONCLUSION:

In this paper, we introduced and studied the concept of soft generalized Haussdorff, soft generalized regular and soft generalized normal in soft grill topological spaces. We established their basic properties and investigated the relationships among these soft generalized separation axioms. Moreover, soft generalized continuous and soft generalized irresolute mapping associated with these spaces were introduced and their fundamental properties were discussed. We hope that the result obtained in this paper will provided a useful foundation for further research on soft grill topological spaces and related areas.

REFERENCE

[1] Banu Pazor Varol and Hails Aygun, On a soft Hausdorff spaces, Comput.Math.Appl.,62(2011), 4058-4067.

[2] Chandramathi. N: A New class of generalized closed sets using grills, Int. J. Math. Archive, 7(7) (2016), 66–71.

[3] Chandramathi. N and Nithya. P, Soft generalized closed sets in soft grill topological spaces, Indian Journal of Natural Sciences, Vol. 15/issue 87, ISSN 0976-0997,86539-86543(2024).

[4] Chandramathi. N and Nithya. P, Soft generalized continuous mapping and irresolute in soft grill topological spaces, Advanced Engineering Science, vol. 57, issue 02,(2025), 291-297.

[5] D. Molodtsov, Soft Set Theory-First Results, Comp. Math. Appl. 37(1999) 19-31.

[6] Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; Soft regularity and normality based on semi open soft sets and soft ideals, Appl. Math. Inf. Sci. Lett., 3 (2) (2015) 47–55.

[7] Kiruthika V, Chandramathi N and Nithya P, soft $\alpha\omega\ddot{\mathrm{I}}_S$- Normal and regular spaces in Soft ideal topological spaces, Kongunadu Research Journal, ISSN 2349-2694, Vol.12(2),2025, pp 19-23.

[8] M. Shabir and M. Naz, On soft topological spaces, Computers and Mathematics with Applications 61 (7) (2011) 1786–1799.

[9] Rodyna A. Hosny and A.M.Abd El-Latif, Soft $\mathcal{G}$ compactness in soft topological spaces, Annals of Fuzzy Mathematics and Informatics Volume x, ISSN: 2287-6235.

[10] Rodyna A. Hosny,Notes on soft perfect sets, International Journal of Math. Analysis, 8(17) (2014) 803-812.

[11] W. Rong, The countabilities of soft topological spaces, Int. J. Comput. Math. Sci. 6(2012) 159-162.

[12] B. Roy and M.N. Mukhreejee, On a typical topology induced by a grill, Soochow J. Math., 33(4)(2007),771-786.

Cite this Article:

Chandramathi, N.; Nithya, P.; Kiruthika, V. (2026). Soft $\mathcal{G}$ Hausdorff, Soft $\mathcal{G}$ Normal and Soft $\mathcal{G}$ Regular Via Soft Grill Topological Spaces. Pi International Journal of Mathematical Sciences, 2(3), 17–25.

Journal URL: https://pijms.com/

DOI: https://doi.org/10.59828/pijms.v2i3.38