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Analytical Study of Distance-Based Topological Indices for the Cluster Product of K_m and P_n

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ABSTRACT

A topological index is an analytically derived numerical quantity that characterizes the structural properties of a graph. These indices play a crucial role in chemical graph theory and quantitative structure–property/activity relationship (QSPR/QSAR) studies. In this paper, we investigate several distance-based topological indices and clusters, including the Wiener index (W), Hyper-Wiener index (WW), Harary index (H), Reciprocal Complementary Wiener index (RCW), Sum Connectivity Index (SCI), Batschelet Index (B), Reverse Wiener index (Λ), Reciprocal Reverse Wiener index (RΛ(G)).

Keywords: Wiener index, Hyper-Wiener Index, Harary Index, Reciprocal Complementary Wiener Index, Sum Connectivity Index, Batschelet Index, Reverse Wiener Index, Reciprocal Reverse Wiener Index and Cluster Product.

Mathematics Subject Classification (MSC) 2020: 05C76, 05C10, 05C38.

1. Introduction

Graph theory, a fundamental area of mathematics, is extensively applied in many scientific and engineering fields. Basically, a graph $G(V,E)$ consists of vertices connected by edges, where vertices represent nodes and edges denote the links between them[1,2]. Edges connected to a vertex are called the degree of the vertex and the shortest length of a connected series of vertices forming a path is called the distance of the graph and is denoted by $d(u,v)$. Topological indices, which are numerical values associated with chemical graphs, correlate with the biological activities and various physical properties of the molecules they represent. [3]. Distance-based topological indices, which incorporate the notion of distance between vertices in a network, are among these indices that are essential for comprehending the structural characteristics of chemical compounds. Total graphs provide a holistic view of chemical network analysis by including both vertex and edge components in a single framework [4].

The fields of graph theory develop various applications in the field of chemistry by using incident or adjacent matrices. In the respective years of 2001 [5] and 2002 [6], Dobrynin and Bonchek studied Wiener index for predicting the molecular ability of a chemical structure. Hosoya and Harary polynomials for graphs are studied by Chen et al. and others for targeting a powerful tool for proving modeling and analyzing molecular structures [7, 8]. [9] contributed to the field with research on distance-based indices in Parikh word representable graphs, while Malik et al. [10, 11] highlighted these indices and related graph entropies in fractal-like molecular graphs.

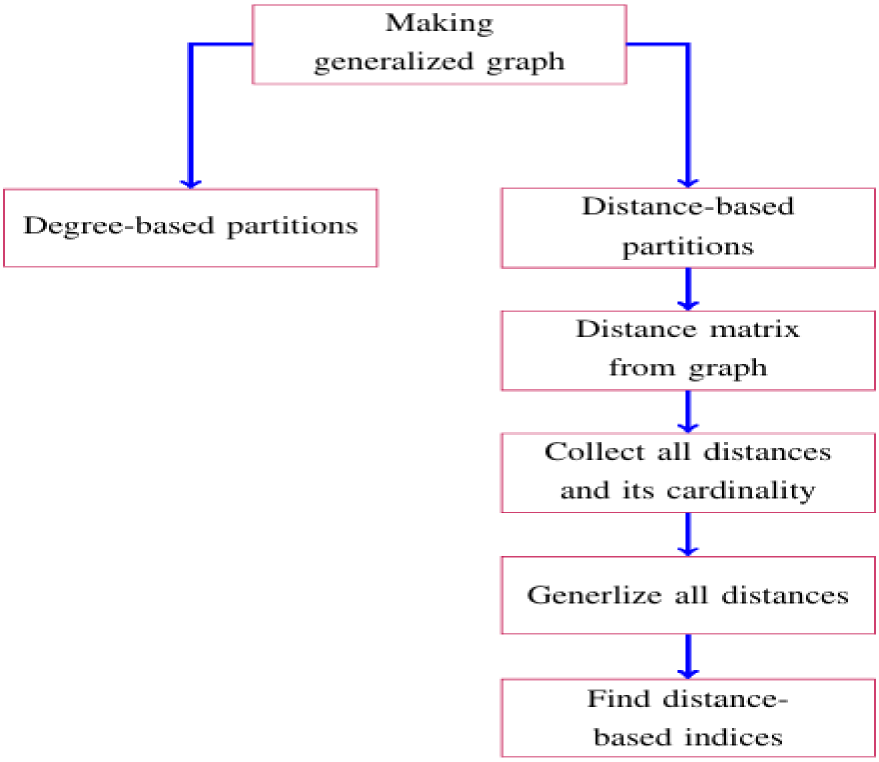
Comparative study of topological indices for total graphs based on distance offers important new information about the fundamental characteristics of chemical structures. This paper explores a number of well-known distance-based indices, such as the hyper-Wiener index, the Harary index, and the Wiener index [12, 13]. In-depth comparisons of these indices are provided in this research, along with information on their advantages, disadvantages, and suitability for various kinds of chemical networks. Our goal in doing this study is to advance the field of chemical graph theory by offering a solid foundation for the use of distance-based topological indices in complete graph analysis. on sequent, this facilitates the forecasting and enhancement of molecular characteristics for many scientific and industrial uses. We compute the following distance-based topological indices shown in Table 1. In this methodology, we first obtain the desired generalized graph where structure of the graph is in accordance with the required size of the graph. After the graph is created, it is necessary to calculate distance matrix which contains the shortest path distance between each pair of identified vertices. Then we create a vector with all possible distances in it and find their frequencies, that is, the number of times each distinct distance appears in the matrix. This step is generalized according to the size of the graph to ensure scalability for any degree of the graph. These collected distances are organized in the tabular form for easier reference as well as for purpose of further computation of the cardinalities. Employing the table of generalized distances, we calculate various distance-based topological indices including the Wiener index, hyper-Wiener index, Harary index, etc. For every index, the distances and the count of the units are substituted in the formulas equal to the indices chosen. This approach gives a method of quantifying the graph through distance-based features in a consistent and generalized manner irrespective of the size of the graph. The flowchart of the methodology is shown in Figure 1.

Only finite, undirected, connected and simple graphs are examined in this study. The numbers of vertices and edges in a graph $G = (V, E)$ are represented by $|V(G)|$ and $|E(G)|$, respectively. If there is no ambiguity in the graph under discussion, we simply denote it by $d(u, v). If\ u, v \in V(G)$, the length of the shortest distance between u and v in G is given by $dG(u, v)$. A vertex u in a graph G has eccentricity $e(u) = max\{d(u, v) : v \in V(G)\}. r = rad(G) = min\{e(v) : v \in V(G)\}$ is the radius(or, more accurately, the diameter) of G $(resp. d = diam(G) = max\{e(v) : v \in V(G)\})$.

Topological indices derive from graph theory, which represents molecules as graphs. Atoms correspond to vertices, whereas bonds correspond to edges. Distance based topological indices are topological indices that are calculated from the distances between vertices (atoms) in a molecular network. These indices concentrate on the graph structure by measuring the distance between atoms, which can offer information about numerous chemical characteristics and behaviors.

Table 1: Distance-based indices

Indices	Notations	Formula
Wiener index [14]	$W(G)$	$W(G) = \sum_{\{u,v \in V(G)\}} d(u,v)$
Hyper-Wiener index [15]	$WW(G)$	$WW(G) = \frac{1}{2} \sum_{\{u,v \in V(G)\}} d(u,v) + d(u,v)^2$
Harary index [16]	$H(G)$	$H(G) = \sum_{\{u,v \in V(G)\}} \frac{1}{d(u,v)}$
Reciprocal Complementary Wiener index [17]	$RCW(G)$	$RCW(G) = \sum_{\{u,v \in V(G)\}} \frac{1}{(d + 1 - d(u,v))}$
Sum Connectivity index [18]	$SCI(G)$	$SCI(G) = \sum_{\{u,v \in V(G)\}} \frac{1}{(d(u,v) + 1)}$
Batschelet index [19]	$B(G)$	$B(G) = \sum_{\{u,v \in V(G)\}} d(u,v)^2$
Reverse Wiener index [20]	$\Lambda(G)$	$\Lambda(G) = \frac{n(n-1)d}{2} - W(G)$
Reciprocal Reverse Wiener index [21]	$R\Lambda(G)$	$R\Lambda(G) = \sum_{\{u,v \in V(G)\}} \frac{1}{(d - d(u,v))}$



A graph is considered complete when each vertex is connected to every other vertex. A complete graph of r vertices is denoted by Kr.

An open walk that does not revisit any vertex is known as a path, often represented by P. A path graph consisting of s vertices is referred to as Ps. In this paper we calculate $W(G), WW(G), H(G), RCW(G), SCI(G), B(G), \wedge(G)$ and $R \wedge (G)$ of cluster product of complete with path graphs.

Definition 1.1. *[22] The cluster of G{H}, formed from two graphs G and H, is defined as the graph constructed by selecting one vertex from G and creating |V(G)| copies of the rooted graph H, with the root of the* i^{th} *vertex in G connected to every vertex in the* j^{th} *copy of H.*

See Figure 2

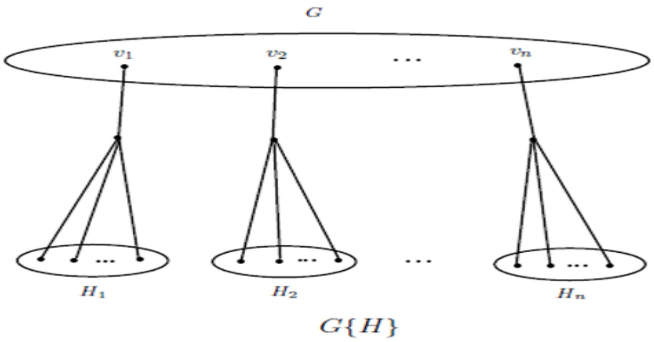


Figure 2: Cluster Product of two graphs G and H

2. Main Results

In this section, we calculate several distance-based topological indices for the cluster product of $Kr\{Ps\}$.

Definition 2.1. The graph that results from taking r vertex of Ps, one vertex of K_r and linking each vertex in the i^{th} vertex of Ps, to every vertex in the j^{th} vertex of K_r is known as the cluster product of complete and path graphs, Kr and Ps.

Here diam(G) = 5 and the distance between any pair of vertices, from $1, 2, \cdots$ $diam(G)$.

The count of vertex pairs at distance 1 is expressed as $\frac{r(r-1)}{2} + 2rs$.

For distance 2, it is $rs(r - 1) + \frac{r(s-1)(s-2)}{2} + rs$.

At distance 3, the count is $\frac{r(r-1)}{2} + rs(r-1)$.

The number of pairs at distance 4 equals $rs(r-1)$.

Lastly, for $r \geq 2$ and $s \geq 2$, the number of vertex pairs at distance 5 is given by $\frac{r(r-1)s^2}{2}$.

These expressions form the basis for deriving subsequent theorems. Then,

Theorem 2.1. Let K_r and P_s be complete and the path graph with $r \geq 2$ and $s \geq 2$. Then

$$(i)\ W(K_r\{P_s\}) = \left(\frac{r(r-1)}{2}\right)[5s^2 + 14s + 8] + r\,[s^2 + s + 2]$$

$$(ii)\ WW(K_r\{P_s\}) = \left(\frac{r(r-1)}{2}\right)[15s^2 + 32s + 13] + r\left[\frac{3s^2 + s + 6}{2}\right]$$

Proof. Let K_r and P_s be complete and the path graph with $r, s \geq 2$, then

(i) By the definition of Wiener index

$$W(K_r\{P_s\}) = \sum d(u,v),\ u,v \in V(G)$$

$$= \left[\frac{r(r-1)}{2} + 2rs\right](1) + \left[r(r-1) + \frac{r(s-1)(s-2)}{2} + rs\right](2)$$
$$+ \left[\frac{r(r-1)(2s+1)}{2}\right](3) + [rs(r-1)]\,(4) + \left[\frac{r(r-1)s^2}{2}\right](5)$$

$$= \frac{r(r-1)}{2} + 2rs + 2r(r-1) + r(s-1)(s-2) + 2rs + \frac{3r(r-1)(2s+1)}{2} + 4rs(r-1)$$
$$+ \frac{5r(r-1)s^2}{2}$$

$$= \left(\frac{r(r-1)}{2}\right)[1 + 4 + 3(2s+1) + 8s + 5s^2] + 4rs + r(s^2 - 3s + 2)$$

$$= \left(\frac{r(r-1)}{2}\right)[5s^2 + 14s + 8] + r[s^2 + s + 2]$$

(ii) By the definition of Hyper-Wiener index

$$WW(K_r\{P_s\}) = \frac{1}{2}\sum[\,d(u,v) + d(u,v)^2]\ , u,v \in V(G)$$

$$= \frac{1}{2}\left\{\left[\frac{r(r-1)}{2} + 2rs\right](1 + 1^2) + \left[r(r-1) + \frac{r(s-1)(s-2)}{2} + rs\right](2 + 2^2)\right.$$
$$\left. + \left[\frac{r(r-1)(2s+1)}{2}\right](3 + 3^2) + [rs(r-1)]\,(4 + 4^2) + \left[\frac{r(r-1)s^2}{2}\right](5 + 5^2)\right\}$$

$$= 1/2\left\{\left[\frac{r(r-1)}{2} + 2rs\right](2) + \left[r(r-1) + \frac{r(s-1)(s-2)}{2} + rs\right](6)\right.$$
$$\left. + \left[\frac{r(r-1)(2s+1)}{2}\right](12) + [rs(r-1)]\,(20) + \left[\frac{r(r-1)s^2}{2}\right](30)\right\}$$

$$= \frac{1}{2}\{r(r-1) + 4rs + 6r(r-1) + 3r(s-1)(s-2) + 6rs + 6r(r-1)(2s+1) + 20rs(r-1)$$
$$+ 15r(r-1)s^2\}$$

$$= \frac{r(r-1)}{2} + 2rs + 3r(r-1) + \frac{3}{2}r(s-1)(s-2) + 3rs + 3r(r-1)(2s+1) + 10rs(r-1)$$
$$+ \frac{15}{2}r(r-1)s^2$$

$$= \left(\frac{r(r-1)}{2}\right)[1 + 6 + 6(2s+1) + 20s + 15s^2] + 5rs + \frac{3}{2}r(s^2 - 3s + 2)$$

$$= \left(\frac{r(r-1)}{2}\right)[7 + 12s + 6 + 20s + 15s^2] + r\left[5s + \frac{3}{2}(s^2 - 3s + 2)\right]$$

$$= \left(\frac{r(r-1)}{2}\right)[15s^2 + 32s + 13] + r\left[\frac{(3s^2 + s + 6)}{2}\right]$$

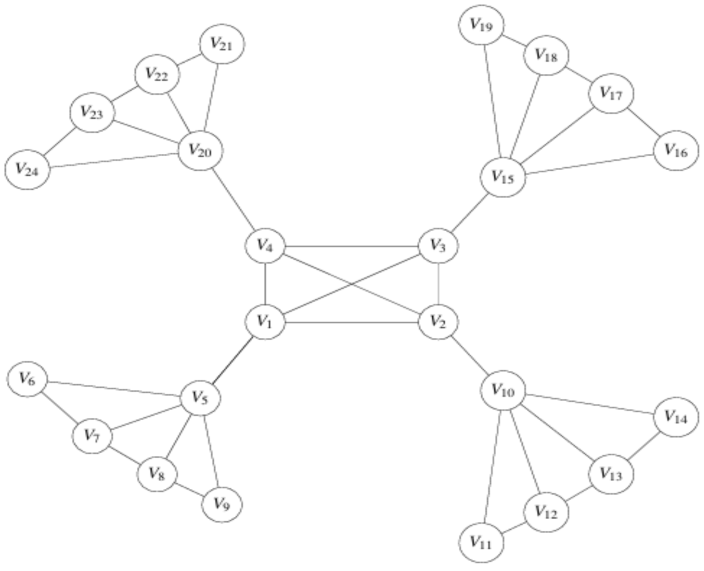


Figure 3: $K_4\{P_4\}$

Theorem 2.2. Let K_r and P_s be complete and the path graph with $r \geq 2$ and $s \geq 2$. Then

$$(i)\ H(K_r\{P_s\}) = \left(\frac{r(r-1)}{2}\right)\left[\frac{(6s^2+35s+70)}{30}\right] + r\left[\frac{(s^2+7s+2)}{4}\right]$$

$$(ii) RCW(K_r\{P_s\}) = \left(\frac{r(r-1)}{2}\right)\left[\frac{(30s^2+50s+31)}{30}\right] + r\left[\frac{(5s^2+11s+10)}{40}\right]$$

Proof. Let K_r and P_s be as specified earlier, then

(i) By the definition of Harary index

$$H(K_r\{P_s\}) = \sum \frac{1}{d(u,v)}\,,\quad u,v \in V(G)$$

$$= \{[r(r-1)/2 + 2rs](1/1) + \left[r(r-1) + \frac{r(s-1)(s-2)}{2} + rs\right]\left(\frac{1}{2}\right)$$

$$+ \left[\frac{r(r-1)(2s+1)}{2}\right]\left(\frac{1}{3}\right) + [rs(r-1)]\left(\frac{1}{4}\right) + \left[\frac{r(r-1)s^2}{2}\right]\left(\frac{1}{5}\right)\}$$

$$= \frac{r(r-1)}{2} + 2rs + \frac{r(r-1)}{2} + \frac{r(s-1)(s-2)}{4} + \frac{rs}{2} + \frac{r(r-1)(2s+1)}{6}$$

$$+ \frac{rs(r-1)}{4} + \frac{r(r-1)s^2}{10}$$

$$= \left(\frac{r(r-1)}{2}\right)\left[1 + 1 + \frac{2s+1}{3} + \frac{s}{2} + \frac{s^2}{5}\right] + r\left[2s + \frac{(s-1)(s-2)}{4}\right]$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{60}{30} + \frac{20s}{30} + \frac{15s}{30} + \frac{6s^2}{30}\right] + r\left[\frac{8s + 2s - s^2 - 3s + 2}{4}\right]$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{6s^2+35s+70}{30}\right] + r\left[\frac{s^2+7s+2}{4}\right]$$

(ii) By the definition Reciprocal Complementary Wiener index

$$RCW(K_r\{P_s\}) = \sum \frac{1}{d+1-d(u,v)},\quad u,v \in V(G)$$

$$= \{\left[\frac{r(r-1)}{2} + 2rs\right]\left(\frac{1}{5}\right) + \left[r(r-1) + \frac{r(s-1)(s-2)}{2} + rs\right]\left(\frac{1}{4}\right)$$

$$+ \left[\frac{r(r-1)(2s+1)}{2}\right]\left(\frac{1}{3}\right) + [rs(r-1)]\left(\frac{1}{2}\right) + \left[\frac{r(r-1)s^2}{2}\right]\left(\frac{1}{1}\right)\}$$

$$= \frac{r(r-1)}{10} + \frac{2rs}{5} + \frac{r(r-1)}{4} + \frac{r(s-1)(s-2)}{8} + \frac{rs}{4} + \frac{r(r-1)(2s+1)}{6}$$

$$+ \frac{rs(r-1)}{2} + \frac{r(r-1)s^2}{2}$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{1}{5}+\frac{1}{2}+\frac{2s+1}{3}+s+s^2\right]+r\left[\frac{2s}{5}+\frac{s^2-3s+2}{8}+\frac{s}{4}\right]$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{7}{10}+\frac{2s}{3}+\frac{1}{3}+s+s^2\right]+r\left[\frac{(5s^2+11s+10)}{40}\right]$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{(30s^2+30s+20s+10+21)}{30}\right]+r\left[\frac{(5s^2+11s+10)}{40}\right]$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{(30s^2+50s+31)}{30}\right]+r\left[\frac{(5s^2+11s+10)}{40}\right]$$

Theorem 2.3. Let K_r and P_s be complete and the path graph with $r \geq 2$ and $s \geq 2$. Then

$$(i)\ SCI(K_r\{P_s\}) = \left(\frac{r(r-1)}{2}\right)\left[\frac{(10s^2+54s+85)}{60}\right]+r\left[\frac{(s^2+5s+2)}{6}\right]$$

$$(ii) B(K_r\{P_s\}) = \left(\frac{r(r-1)}{2}\right)[\,25s^2+50s+18\,]+r\,[\,2s^2+4\,]$$

Proof. Under the assumption that K_r and P_s as specified earlier, then

(i) By the definition of Sum Connectivity index

$$SCI(K_r\{P_s\}) = \sum \frac{1}{d(u,v)+1}\ ,\ \ u,v \in V(G)$$

$$= \left\{\left[\frac{r(r-1)}{2}+2rs\right]\left(\frac{1}{1+1}\right)+\left[r(r-1)+\frac{r(s-1)(s-2)}{2}+rs\right]\left(\frac{1}{2+1}\right)\right.$$

$$\left.+\left[\frac{r(r-1)(2s+1)}{2}\right]\left(\frac{1}{3+1}\right)+[\,rs(r-1)]\left(\frac{1}{4+1}\right)+\left[\frac{r(r-1)s^2}{2}\right]\left(\frac{1}{5+1}\right)\right\}$$

$$= \frac{r(r-1)}{4}+rs+\frac{r(r-1)}{3}+\frac{r(s-1)(s-2)}{6}+\frac{rs}{3}+\frac{r(r-1)(2s+1)}{8}$$

$$+\frac{rs(r-1)}{5}+\frac{r(r-1)s^2}{12}$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{1}{2}+\frac{2}{3}+\frac{2s+1}{4}+\frac{2s}{5}+\frac{s^2}{6}\right]+r\left[s+\frac{s}{3}+\frac{s^2-3s+2}{6}\right]$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{(30+40+30s+15+24s+10s^2)}{60}\right]+r\left[\frac{(s^2-3s+2+2s+6s)}{6}\right]$$

$$= \left(\frac{r(r-1)}{2}\right)\left[\frac{(10s^2+54s+85)}{60}\right]+r\left[\frac{(s^2+5s+2)}{6}\right]$$

(ii) By the definition of Batschelet index

$$B(K_r\{P_s\}) = \sum d(u,v)^2\ ,\ \ u,v \in V(G)$$

$$= \left[\frac{r(r-1)}{2}+2rs\right](1^2)+\left[r(r-1)+\frac{r(s-1)(s-2)}{2}+rs\right](2^2)$$

$$+\left[\frac{r(r-1)(2s+1)}{2}\right](3^2)+[\,rs(r-1)](4^2)+\left[\frac{r(r-1)s^2}{2}2\right](5^2)$$

$$= \frac{r(r-1)}{2}+2rs+4r(r-1)+2r(s-1)(s-2)+4rs+\frac{9r(r-1)(2s+1)}{2}+16rs(r-1)$$

$$+\frac{25r(r-1)s^2}{2}$$

$$= \left(\frac{r(r-1)}{2}\right)[\,1+8+18s+9+32s+25s^2]+r\,[\,2s^2+2s-6s+4+4s\,]$$

$$= \left(\frac{r(r-1)}{2}\right)[\,25s^2+50s+18\,]+r\,[\,2s^2+4\,]$$

Theorem 2.4. Let the parameters K_r and P_s be as stated above, then

$(i)\ \Lambda(K_r\{P_s\}) = \frac{r}{2}\{5(s+1)[r(s+1)-1] - [(r-1)(5s^2+14s+8) - 2(s^2+s+2)]\}$

$(ii)\ R\Lambda(K_r\{P_s\}) = r(r-1)/2\,[(36s+17)/12] + r\left[\frac{s^2+2s+2}{6}\right]$

Proof. Under the assumption that K_r and P_s be as defined above, then

(i) By the definition of Reverse Wiener index,

$$\Lambda(K_r\{P_s\}) = \frac{n(n-1)d}{2} - W(G)$$

$$= (r+rs)\big((r+rs)-1\big)\cdot\frac{5}{2} - \frac{r(r-1)}{2}(5s^2+14s+8) - r(s^2+s+2)$$

$$= \frac{5}{2}[r^2 + r^2s - r + r^2s + r^2s^2 - rs] - \frac{r(r-1)}{2}(5s^2+14s+8) - r(s^2+s+2)$$

$$= 5\left[\frac{[r^2(1+2s+s^2) - r(s+1)]}{2}\right] - \frac{r(r-1)}{2}(5s^2+14s+8) - r(s^2+s+2)$$

$$= \frac{r}{2}\{5[r(s^2+2s+1) - (s+1)] - [(r-1)(5s^2+14s+8) - 2(s^2+s+2)]\}$$

$$= \frac{r}{2}\{5(s+1)[r(s+1) - 1] - [(r-1)(5s^2+14s+8) - 2(s^2+s+2)]\}$$

(ii) By the definition of Reciprocal Reverse Wiener index,

$$R\Lambda(K_r\{P_s\}) = \sum_{\{u,v\in V(G)\}} \frac{1}{d - d(u,v)}$$

$$= \left[\frac{r(r-1)}{2} + 2rs\right]\left(\frac{1}{5-1}\right) + \left[r(r-1) + \frac{r(s-1)(s-2)}{2} + rs\right]\left(\frac{1}{5-2}\right)$$
$$+ \left[\frac{r(r-1)(2s+1)}{2}\right]\left(\frac{1}{5-3}\right) + [rs(r-1)]\left(\frac{1}{5-4}\right) + \left[\frac{r(r-1)s^2}{2}\right]\left(\frac{1}{5-5}\right)$$

$$= \frac{r(r-1)}{8} + \frac{rs}{2} + \frac{r(r-1)}{3} + \frac{r(s-1)(s-2)}{6} + \frac{rs}{3} + \frac{r(r-1)(2s+1)}{4} + rs(r-1)$$

$$= \frac{r(r-1)}{2}\left[\frac{1}{4} + \frac{2}{3} + \frac{2s+1}{2} + 2s\right] + r\left[\frac{s}{2} + \frac{s^2-3s+2}{6} + \frac{s}{3}\right]$$

$$= \frac{r(r-1)}{2}\left[\frac{36s+17}{12}\right] + r\left[\frac{s^2+2s+2}{6}\right]$$

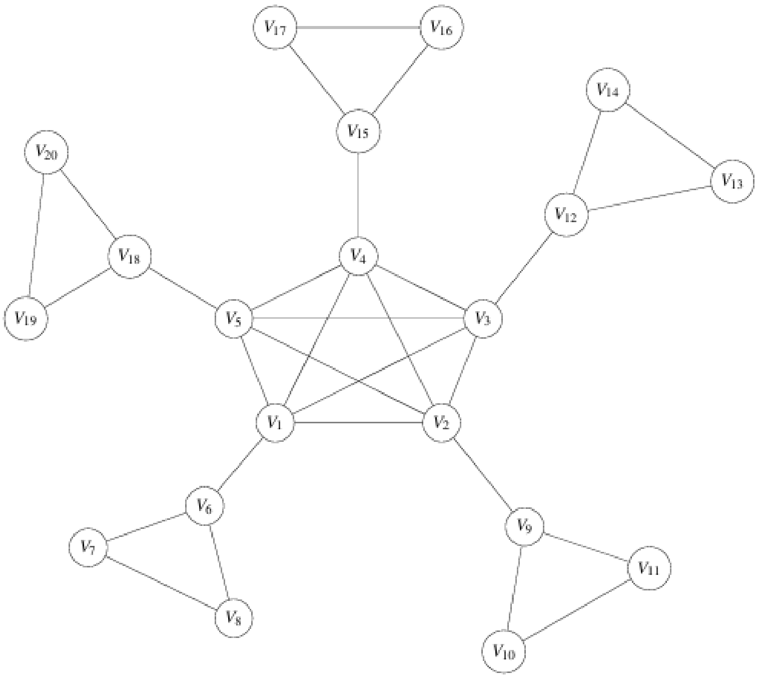


Figure 4: $K_5\{P_2\}$

References

[1]. D. B. West, *Introduction to Graph Theory* (Upper Saddle River, NJ: Prentice-Hall, 1996).

[2]. P. Et al, *“Distance Based Topological Indices and Regular Graphs”*, Turkish Journal of Computer and Mathematics Education 12, no. 3 (2021): 5191–5196,

[3]. K. Pattabiraman, *“Degree and Distance Based Topological Indices of Graphs,”* Electronic Notes in Discrete Mathematics 63, no. 5 (2017): 145–159,

[4]. S. Stevanovic and D. Stevanovic, *“On Distance-Based To pological Indices Used in Architectural Research,”* MATCH Commun. Math. Comput. Chem. 79, no.7 (2018): 659–683.

[5]. A. A. Dobrynin, R. Entringer, and I. Gutman, *“Wiener Index of Trees: Teory and Applications,”* Acta Applicandae Mathematica 66, no. 3 (2001): 211–249,

[6]. D. Bonchev, *“Te Wiener Number-Some Applications and NewDevelopments,”* in Topology in Chemistry (Sawston, UK: Woodhead Publishing, 2002), 58–88.

[7]. L. Chen,A.Mehboob,H.Ahmad,W.Nazeer,M.Hussain,and M. R. Farahani, *“Hosoya and Harary Polynomials of and,”* Discrete Dynamics in Nature and Society 20, no. 19 (2019): 23–36.

[8]. G. Zhou, H. Li, R. Song, Q. Wang, J. Xu, and B. Song, *“Orthorectifcation of Fisheye Image Under Equidistant Projection Model,”* Remote Sensing 14, no. 17 (2022): 4175,

[9]. N. Tomas, L. Mathew, S. Sriram, A. K. Nagar, and K. G. Subramanian, “Certain Distance Based Topological Indices of Parikh Word Representable Graphs,” Journal of Mathematics 2021, no. 1 (2021): 1–7,

[10]. M. A. Malik, M. Imran, and M. Adeel, *“On Distance-Based Topological Indices and Co indices of Fractal-Type Molecular Graphs and Teir Respective Graph Entropies,”* PLoS One 18, no. 11 (2023): e0290047–e0290059,

[11]. Q. Chen, L. Yang, Y. Zhao, Y. Wang, H. Zhou, and X. Chen, *“Shortest Path in LEO Satellite Constellation Networks: An Explicit Analytic Approach,”* IEEE Journal on Selected Areas in Communications 42, no. 5 (2024): 1175–1187,

[12]. M. K. Jamil, *“Distance-based Topological Indices and Double Graph,”* Iranian Journal of Mathematical Chemistry 8, no. 1 (2017): 83–91.

[13]. D.Wan,J. Yang,T.Zhang,andY.Xiong, *“A Novel Method to Identify Influential Nodes Based on Hybrid Topology Structure,”* Physical Communication 58 (2023)

[14]. H. Wiener, *“Structural Determination of Parafn Boiling Points,”* Journal of the American Chemical Society 69, no. 1 (1947): 17–20,

[15]. M. Randi´c, *“Novel Molecular Descriptor for Structure—Property Studies,”* Chemical Physics Letters 211, no. 4-5 (1993): 478–483,

[16]. D. Plavˇsic, S. Nikoli ´ c, N. Trinajsti ´ c, and Z. Mihali ´ c, *“On the ´ Harary Index for the Characterization of Chemical Graphs,”* Journal of Mathematical Chemistry 12, no. 1 (1993): 235–250,

[17]. Nasiri, R., Nakhaei, A., Shojaeifard, A. R. (2021). *"The reciprocal complementary Wiener number of graph operations".* Kragujevac Journal of Mathematics, 45(1), 139-154.

[18]. Randic, M. (1975). *“On characterization of molecular branching.”* Journal of the American Chemical Society, 97(23),6609-6615.

[19]. Batschelet, E. (1981).*”Recent advances in graph theory.”* Proceedings of the Second International Conference on Graph Theory, 13-23.

[20]. A.T.Balaban, D.Mills, O.Ivanciuc, S.C.Basak, *"Reverse Wiener indices",* Croat.Chem.Acta. 73 (2000) 923-941.

[21]. Zhou, B., Yang, Y., Trinajstic, N. (2010).´ *"On reciprocal reverse Wiener index".* Journal of mathematical chemistry, 47, 201-209.

[22]. G. Indulal, D. Stevanovic, (2015). The distance spectrum of corona and cluster of two graphs.´ AKCE International Journal of Graphs and Combinatorics, 12(2-3), 186-192.

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