Pi International Journal of Mathematical Sciences
Volume 2 | Issue 4 | August 2026 | ISSN: 3107-9830 (Online)
Available online at: https://pijms.com/
DOI: https://doi.org/10.59828/pijms.v2i4.42

Stability Behavior of Bianchi Type I Cosmological Models with Variable G under a Creation Field

Apurav Radhakrishna Gupta
Department of Mathematics, L.P.G. Art's and Science College Shirpur (Jain), Washim, Maharashtra, India
Corresponding Author E-mail: apuravgupta2011@gmail.com

Received: 17 July 2026 | **Accepted:** 12 August 2026 | **Published:** 27 August 2026

Abstract

This paper introduces a modification of Einstein's field equations through the incorporation of a massless scalar field, referred to as the C-field. It investigates a cosmological model with a variable gravitational constant (G) within the framework of Bianchi type I space-time, taking into account the presence of a creation field. To construct a deterministic model, the study assumes $G = V^n$ *where* $V = DEF$ *represents the volumetric function, in accordance with the principles of steady-state cosmology and the creation field. The model satisfies conservation laws, demonstrates an increasing creation field over time, avoids singularities, and offers a natural explanation for inflationary dynamics and isotropisation. Graphical analyses are employed to illustrate the evolution of key cosmological parameters. Furthermore, the model explores acceleration properties, revealing an expanding and accelerating universe consistent with observational evidence. The universe is assumed to be filled with a perfect fluid, characterised by the equation* $P = \gamma\rho$ *, where* γ *a constant within the range is* $\gamma \epsilon\ [0,1]$*. Different values of* γ *correspond to distinct cosmological scenarios:* $\gamma = 0$ *represents a dust-dominated universe,* $\gamma = 1$ *corresponds to the Zel'dovich universe or stiff matter, and* $\gamma = \frac{1}{3}$ *describes a radiation-dominated (non-relativistic) fluid. The results for these scenarios are analysed and discussed in detail.*

Keywords: *Bianchi type I Cosmology, C-field, Perfect fluid conditions, Variable G.*
Mathematics Subject Classification (MSC) 2020*: 83C15, 83F05, 83C55, 83C75.*

1. Introduction

In modern cosmology, the discovery by Riess et al. [1] established that the present universe is both expanding and accelerating. Perlmutter et al. [2] further confirmed this through high-redshift supernova experiments, validating the acceleration and expansion during late cosmic epochs. To account for these phenomena, several modifications to general relativity have been proposed. Brans et al. [3] introduced a scalar-tensor theory involving a coupling between a tensor field and a scalar field ϕ, where the scalar field has the dimension of G−1, with G denoting the gravitational constant. Subsequently, Saez et al. [4] developed a new scalar–tensor theory incorporating a dimensionless scalar field, providing a robust framework for the description of weak fields. A model capable of explaining the creation of positive-energy matter, whilst preserving the principle of energy conservation, requires a corresponding negative-energy mode, which offers a natural mechanism for matter formation. Hoyle and Narlikar [5] developed a field-theoretic framework to address this by introducing a massless and chargeless scalar field, termed the creation field (C-field). Unlike big bang cosmology, C-field cosmology avoids singularities. These models have attracted attention because they resolve several key issues in big bang cosmology, including the singularity, horizon, and flatness problems. Alternative approaches to these challenges include (i) quantum gravity models and (ii) inflationary cosmological models. However, due to the absence of a fully developed quantum theory of gravity and the inability of inflationary models to completely resolve the singularity problem, the negative-energy field within the C-field framework offers a compelling alternative.

Gravitation plays a dominant role on large scales because of the limited range of strong and weak forces and the diminished effect of the electromagnetic force owing to the overall neutrality of matter, as noted by Dicke and Peebles [6]. For a model to explain the formation of positive-energy matter without violating energy conservation, it must incorporate a mechanism involving a negative-energy mode. Various quantum gravitational models adopt this approach, utilizing a negative-energy mode arising from gravity's scale degree of freedom. This provides a natural mechanism for matter creation, as classical singularity theorems no longer hold when the positivity of energy density is not assured. Hoyle and Narlikar [7] proposed a theoretical approach based on a massless, chargeless scalar field to account for matter creation. Narlikar and Padmanabhan [8] explored solutions to Einstein's field equations incorporating radiation and a massless scalar creation field with negative energy as sources. They demonstrated that such cosmological models satisfy observational requirements, provide a viable alternative to the big bang model, and resolve singularity and particle horizon issues, whilst also addressing the flatness problem. The concept of the phantom field is regarded as a revival of the C-field, as discussed by Singh et al. [9], Giacomini and Lara [10], and Vishwakarma and Narlikar [11], who analysed models of repulsive gravity associated with matter creation. More recently, Bali and Tikekar [12], as well as Bali and Kumawat [13], investigated C-field cosmological models considering dust distributions in FRW space-time with a variable gravitational constant.

Several researchers have also studied cosmological models incorporating variations in the gravitational constant (G) and the cosmological constant (Λ) within different theoretical frameworks. Saraf [14] examined the Bianchi Type I cosmological model for dust distributions with variable G and (Λ). Dagwal and Pawar [15] explored two-fluid cosmological models with variable G and (Λ) in the framework of general relativity. Alfedeel [16] investigated generalised solutions of the Bianchi Type I model with time-varying G and (Λ). Singh, Baro, and Meitei [17] considered higher-dimensional Bianchi Type I cosmological models with massive strings in general relativity. Tiwari, Alfedeel, and collaborators [18] studied cosmological models with time-dependent (Λ), G, and viscous fluid in the framework of general relativity. Finally, Bali and Saraf [19] analysed cosmological models with a variable (Λ) term in Bianchi Type III space-time within the creation field framework.

In **Section 2**, we define the Hoyle and Narlikar modification of Einstein's field equations through the introduction of the C-field. In **Section 3**, we formulate the field equations for a barotropic perfect fluid with a variable gravitational constant GGG within the framework of the Bianchi Type I cosmological model. **Section 4** characterised different types of universes: $\gamma = 0$ corresponds to a dust-dominated universe, $\gamma = 1$ represents the Zeldovich universe or stiff matter, and $\gamma = \frac{1}{3}$ indicates a radiation-dominated (non-relativistic) fluid. In **Section 5**, we analyzed the standard stability function of the model. Finally, **Section 6**, we summarized our findings and discuss their broader implications.

2. Hoyle Narlikar Theory

Hoyle and Narlikar [5] modified Einstein's field equation through the introduction of the C-field as follows:"

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G\left[m_{T_{ij}} + C_{T_{ij}}\right], \tag{2.1}$$

where the first term on RHS $m_{T_{ij}}$ refer to matter tensor and $C_{T_{ij}}$ refer the C-field.

Here the matter tensor defined as:

$$m_{T_{ij}} = dia(\rho, -P, -P, -P), \quad (2.2)$$

and

$$C_{T_{ij}} = -f\left(C_i\, C_j - \frac{1}{2} g_{ij}\, C^k\, C_k\right), \quad (2.3)$$

where $f > 0, C_i = \frac{dC}{dx^i}$,

Here, C represents the speed of light. Since $T^{44} < 0$, the C-field exhibits a negative energy density, leading to a repulsive gravitational field that drives the expansion of the universe.

3. Metric and Field Equations

Bianchi type I metric form as:

$$ds^2 = dt^2 - D^2 dx^2 - E^2\, dy^2 - F^2 dz^2\,, \quad (3.1)$$

here D, E and F are only space-time function.

Here the matter tensor defined as:

$$m_{T_{ij}} = dia(\rho, -P, -P, -P). \quad (3.2)$$

The field equation reduced to:

$$\left[\frac{\ddot{E}}{E} + \frac{\ddot{F}}{F} + \frac{\dot{E}\dot{F}}{EF}\right] = 8\pi G\,\left(-P + \frac{1}{2}\, f\, \dot{C}^2\right). \quad (3.3)$$

$$\left[\frac{\ddot{D}}{D} + \frac{\ddot{F}}{F} + \frac{\dot{D}\dot{F}}{DF}\right] = 8\pi G\,\left(-P + \frac{1}{2}\, f\, \dot{C}^2\right). \quad (3.4)$$

$$\left[\frac{\ddot{D}}{D} + \frac{\ddot{E}}{E} + \frac{\dot{D}\dot{E}}{DE}\right] = 8\pi G\,\left(-P + \frac{1}{2}\, f\, \dot{C}^2\right). \quad (3.5)$$

$$\left[\frac{\dot{D}\dot{E}}{DE} + \frac{\dot{E}\dot{F}}{EF} + \frac{\dot{D}\dot{F}}{DF}\right] = 8\pi G\,\left(\rho - \frac{1}{2}\, f\, \dot{C}^2\right). \quad (3.6)$$

An additional equation describing the time variation of G is derived from the divergence of the Einstein tensor, as follows:

$\left[R_{ij} - \frac{1}{2} R g_{ij}\right]_{;j} = 0$, this leads to:

$$8\pi G\left[m_{T_{ij}} + c_{T_{ij}}\right]_{;j} = 0. \quad (3.7)$$

$$8\pi G\,\left(\rho - \frac{1}{2}\, f\, \dot{C}^2\right) + 8\pi G\left(\dot{\rho} + (P + \rho)\frac{\dot{V}}{V} - f\,\dot{C}\left(\ddot{C} + \frac{\dot{V}}{V}\dot{C}\right)\right) = 0, \quad (3.8)$$

where $V = DEF$,

from Eq. (3.8) assume that:

$$\dot{\rho} + (P + \rho)\frac{\dot{V}}{V} - f\,\dot{C}\left(\ddot{C} + \frac{\dot{V}}{V}\dot{C}\right) = 0, \quad (3.9)$$

i.e. $\dot{\rho} + (P + \rho)\frac{\dot{V}}{V} = f\,\dot{C}\left(\ddot{C} + \frac{\dot{V}}{V}\dot{C}\right)$, (3.10)

thus Eq. (3.9) can be written as:

$$\frac{d}{dV}(V\rho) + P = f\,\dot{C}\,\frac{d}{dV}(V\dot{C}). \quad (3.11)$$

To determine a unique solution, it is essential to define the rate at which matter-energy is generated at the cost of negative energy of C-field. Assume that, at any given time, the rate of matter-energy density creation is directly relative to the existing energy density of the C-field. Specifically, the rate at which matter-energy density is created per unit proper volume can be written as:

$$\frac{d}{dV}(V\rho) + P = \alpha^2\,\dot{C}^2 = \alpha^2\, g^2(V), \quad (3.12)$$

where α is a constant, and the following definition is used:

$\dot{C}(V) = g(V)$,

Now using the equation (3.11) and (3.12) yielded as:

$$\frac{d}{dV}(V\rho) + P = f\,g\,\frac{d}{dV}(Vg), \tag{3.13}$$

on integrates provided as:

$$g(V) = A_1 V^{\left(\alpha^2/_f - 1\right)}, \tag{3.14}$$

where A_1 is arbitrary constant.

Assuming the matter equation of state $P = \gamma\rho$, from equations (3.12) and (3.14) obtained as:

$$\frac{d}{dV}(V\rho) + \gamma\rho = \alpha^2\, {A_1}^2\, V^{2\left(\alpha^2/_f - 1\right)}, \tag{3.15}$$

after solving,

$$\rho = \frac{\alpha^2\, {A_1}^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)}\, V^{2\left(\alpha^2/_f - 1\right)}. \tag{3.16}$$

Subtract equation (3.3) from equation (3.4) obtained as:

$$\frac{\ddot{D}}{D} - \frac{\ddot{E}}{E} + \left[\frac{\dot{D}}{D} - \frac{\dot{E}}{E}\right]\frac{\dot{F}}{F} = 0,$$

after solving,

$$\frac{d}{dt}\left[\frac{\dot{D}}{D} - \frac{\dot{E}}{E}\right] = -\left[\frac{\dot{D}}{D} - \frac{\dot{E}}{E}\right]\frac{\dot{V}}{V}, \tag{3.17}$$

on integrating,

$$\frac{D}{E} = d_1\, exp\left\{x_1 \int \frac{dt}{V}\right\}, \tag{3.18}$$

similarly, subtract Eq. (3.3) from (3.5) and (3.4) from (3.5) obtained as:

$$\frac{D}{F} = d_2\, exp\left\{x_2 \int \frac{dt}{V}\right\}, \tag{3.19}$$

$$\frac{E}{F} = d_3\, exp\left\{x_3 \int \frac{dt}{V}\right\}, \tag{3.20}$$

where d_1, d_2, d_3, x_1, x_2 *and* x_3 are integral constant and $d_2 = d_1 d_3$ and $x_2 = x_1 + x_3$.

After solving Eq. (3.18), (3.19), and (3.20) yielded as:

$$D = K_1 V^{1/3}\, exp\left\{X_1 \int \frac{dt}{V}\right\},\; E = K_2 V^{1/3}\, exp\left\{X_2 \int \frac{dt}{V}\right\},$$

$$\text{and } F = K_3 V^{1/3}\, exp\left\{X_3 \int \frac{dt}{V}\right\}, \tag{3.21}$$

where $K_1 K_2 K_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Now by solving Eq. (3.3) + (3.4) + (3.5) + 3(3.6) found as:

$$\left[\frac{\ddot{D}}{D} + \frac{\ddot{E}}{E} + \frac{\ddot{F}}{F}\right] + 2\left[\frac{\dot{D}\dot{E}}{DE} + \frac{\dot{E}\dot{F}}{EF} + \frac{\dot{D}\dot{F}}{DF}\right] = 12\pi G(-P + \rho). \tag{3.22}$$

Now using Eq. (3.9) $V = DEF$ got as:

$$\frac{\ddot{V}}{V} = \left[\frac{\ddot{D}}{D} + \frac{\ddot{E}}{E} + \frac{\ddot{F}}{F}\right] + 2\left[\frac{\dot{D}\dot{E}}{DE} + \frac{\dot{E}\dot{F}}{EF} + \frac{\dot{D}\dot{F}}{DF}\right], \tag{3.23}$$

using Eq. (3.22), (3.23) and $P = \gamma\rho$ found as:

$$\frac{\ddot{V}}{V} = 12\pi G(-\gamma\rho + \rho),$$

$$\frac{\ddot{V}}{V} = 12\pi G\rho(1 - \gamma), \tag{3.24}$$

using $G = V^n$,

$$\frac{\ddot{V}}{V} = 12\pi\rho(1 - \gamma)V^n, \tag{3.25}$$

using $\rho = \frac{\alpha^2\, {A_1}^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)}\, V^{2\left(\frac{\alpha^2}{f} - 1\right)}$,

$$\frac{\ddot{V}}{V} = \frac{12\pi(1-\gamma)\alpha^2\, {A_1}^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)}\, V^{2\left(\frac{\alpha^2}{f} + n - 1\right)},$$

assume $A_2 = \frac{12\pi(1-\gamma)\alpha^2 A_1{}^2}{\left(2\frac{\alpha^2}{f}-1+\gamma\right)}$,

$\frac{\ddot{V}}{V} = A_2\, V^{2\left(\frac{\alpha^2}{f}+n-1\right)}$,

after solving $\frac{dV}{dt} = \left[\frac{A_2}{\left(\frac{\alpha^2}{f}+n-\frac{1}{2}\right)}\right]^{\frac{1}{2}} V^{\left(\frac{\alpha^2}{f}+n-\frac{1}{2}\right)}$,

assume $A_3 = \left[\frac{A_2}{\left(\frac{\alpha^2}{f}+n-\frac{1}{2}\right)}\right]^{\frac{1}{2}}$,

$\frac{dV}{dt} = A_3\, V^{\left(\frac{\alpha^2}{f}+n-\frac{1}{2}\right)}$,

after solving $V = \left[A_3\left(\frac{3}{2}-\frac{\alpha^2}{f}-n\right)t\right]^{\frac{2f}{(3f-2\alpha^2-2nf)}}$,

assume $A_4 = \left[A_3\left(\frac{3}{2}-\frac{\alpha^2}{f}-n\right)\right]^{\frac{2f}{(3f-2\alpha^2-2nf)}}$, (3.26)

$V = A_4\, t^{\left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]}$. (3.27)

Now solving the Eq. (3.14) and (3.27) found that:

$g(V) = A_1 A_4{}^{\left(\frac{\alpha^2}{f}-1\right)}\, t^{\left[\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}$, (3.28)

now using

$\dot{C}(V) = g(V)$,

$\dot{C}(V) = A_1 A_4{}^{\left(\frac{\alpha^2}{f}-1\right)}\, t^{\left[\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}$,

after solving $C = \frac{A_1 A_4{}^{\left(\frac{\alpha^2}{f}-1\right)}(3f-2\alpha^2-2nf)}{f(1-2n)}\, t^{\left[\frac{f(1-2n)}{(3f-2\alpha^2-2nf)}\right]}$,

assume that $A_5 = \frac{A_1 A_4{}^{\left(\frac{\alpha^2}{f}-1\right)}(3f-2\alpha^2-2nf)}{f(1-2n)}$,

$C = A_5\, t^{\left[\frac{f(1-2n)}{(3f-2\alpha^2-2nf)}\right]}$, (3.29)

substitute (3.27) value in Eq. (3.16) obtained as:

$\rho = \frac{\alpha^2 A_1{}^2 A_4{}^{2\left(\frac{\alpha^2}{f}-1\right)}}{\left(\frac{2\alpha^2}{f}-1+\gamma\right)}\, t^{\left[\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}$,

assume that $A_6 = \frac{\alpha^2 A_1{}^2 A_4{}^{2\left(\frac{\alpha^2}{f}-1\right)}}{\left(\frac{2\alpha^2}{f}-1+\gamma\right)}$,

$\rho = A_6\, t^{\left[\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}$. (3.30)

Now we know that $P = \gamma\rho$ obtained as:

$P = \gamma A_6\, t^{\left[\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}$. (3.31)

Now calculate

$\int \frac{dt}{V} = \int \frac{dt}{A_4\, t^{\left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]}}$,

after solving $\int \frac{dt}{V} = \frac{(3f-2\alpha^2-2nf)}{A_4\,(f-2\alpha^2-2nf)}\, t^{\left[\frac{(f-2\alpha^2-2nf)}{(3f-2\alpha^2-2nf)}\right]}$, assume that $A_7 = \frac{(3f-2\alpha^2-2nf)}{A_4\,(f-2\alpha^2-2nf)}$,

$$\int \frac{dt}{V} = A_7\, t^{\left[\frac{(f-2\alpha^2-2nf)}{(3f-2\alpha^2-2nf)}\right]}. \tag{3.32}$$

Now substitute the value V form (3.27) and $\int \frac{dt}{V}$ from (3.32) in equation (3.21) obtained as:

$$D = K_1\, A_4^{\frac{1}{3}}\, t^{\left[\frac{2f}{3(3f-2\alpha^2-2nf)}\right]}\, exp\left\{X_1\, A_7\, t^{\left[\frac{(f-2\alpha^2-2nf)}{(3f-2\alpha^2-2nf)}\right]}\right\},$$

$$E = K_2\, A_4^{\frac{1}{3}}\, t^{\left[\frac{2f}{3(3f-2\alpha^2-2nf)}\right]}\, exp\left\{X_2\, A_7\, t^{\left[\frac{(f-2\alpha^2-2nf)}{(3f-2\alpha^2-2nf)}\right]}\right\},$$

$$F = K_3\, A_4^{\frac{1}{3}}\, t^{\left[\frac{2f}{3(3f-2\alpha^2-2nf)}\right]}\, exp\left\{X_3\, A_7\, t^{\left[\frac{(f-2\alpha^2-2nf)}{(3f-2\alpha^2-2nf)}\right]}\right\}. \tag{3.33}$$

where $K_1K_2K_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Physical Properties:

Average Scale Factor (Hubble Parameter):

$$H = \frac{1}{3}\sum_{i=0}^{3} H_i,$$

$$H = \frac{1}{3}\left[\frac{\dot{D}}{D} + \frac{\dot{E}}{E} + \frac{\dot{F}}{F}\right], \tag{3.34}$$

using the Eq. (3.33) in Eq. (3.34) obtained as:

$$H = \left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \tag{3.35}$$

Expansion Scalar:

$$\theta = 3H \tag{3.36}$$

Using the value of H in Eq. (3.36) yielded as:

$$\theta = \left[\frac{6f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \tag{3.37}$$

Anisotropic Parameter:

$$A = \frac{1}{3}\sum_{i=0}^{3}\left[\frac{\Delta H_i}{H}\right]^2,$$

$$A = \frac{1}{3}\,\frac{(\Delta H_1)^2+(\Delta H_2)^2+(\Delta H_3)^2}{H^2},$$

$$A = \frac{1}{3}\left({X_1}^2 + {X_2}^2 + {X_3}^2\right){A_7}^2\left(\frac{(f-2\alpha^2-2nf)}{2f}\right)^2 t^{\left[\frac{(2f-4\alpha^2-4nf)}{(3f-2\alpha^2-2nf)}\right]},$$

assume that $X^2 = \left({X_1}^2 + {X_2}^2 + {X_3}^2\right)$ then:

$$A = \frac{1}{12}\, X^2\, {A_7}^2\, \frac{\left(f-2\alpha^2-2nf\right)^2}{f^2}\, t^{\left[\frac{(2f-4\alpha^2-4nf)}{(3f-2\alpha^2-2nf)}\right]}. \tag{3.38}$$

Shear Scalar:

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=0}^{3}\Delta {H_i}^2 - 3H^2\right),$$

$$\sigma^2 = \frac{1}{2}\, AH^2,$$

$$\sigma^2 = \frac{1}{2}X^2\, {A_7}^2\, \frac{\left(f-2\alpha^2-2nf\right)^2}{(3f-2\alpha^2-2nf)^2}\, t^{\left[\frac{-4f}{(2f-2\alpha^2-2nf)}\right]}. \tag{3.39}$$

The deceleration parameter q:

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1,$$

Substitute the value of H in Eq. (3.35) found as:

$$q = \frac{d}{dt}\left(\frac{(3f-2\alpha^2-2nf)}{2f}\, t\right) - 1,$$

$$q = \frac{1}{2} - \frac{(\alpha^2+nf)}{f} = -\text{ve} \quad , \tag{3.40}$$

When $\lim\limits_{t\to\infty} \frac{\sigma}{\theta} \neq 0.$

4. Characterization different types of universes

Case I: For $\gamma = 0$ represents a dust-dominated universe:

Spatial Volume V:

Now calculate $g(V)$ using Eq. (3.28),

$$g(V) = A_1 A_4^{\left(\frac{\alpha^2}{f}-1\right)}\; t^{\left[\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]},$$

substitute the value of A_4 from Eq. (3.26) found as:

$$g(V) = A_1 \left[\frac{\sqrt{3\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(2\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}}\; t^{\left[\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}. \tag{4.1}$$

Now calculate C using Eq. (3.29),

$$C = \frac{A_1(3f-2\alpha^2-2nf)}{f(1-2n)}\left[\frac{\sqrt{3\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(2\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}}\; t^{\left[\frac{f(1-2n)}{(3f-2\alpha^2-2nf)}\right]}.$$

$$V = \left[\frac{\sqrt{3\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(2\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}}\; t^{\left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]} \tag{4.2}$$

Therefore, assuming that $3f - 2\alpha^2 - 2nf > 0 \; and \; \alpha^2 < f$, the spatial volume tends to infinity as $t \to \infty$, implying that the Universe continues to expand at late cosmic times.

Energy density ρ:

Now calculate ρ using Eq. (3.30),

$$\rho = \frac{\alpha^2\, {A_1}^2 f}{(2\alpha^2-f)}\left[\frac{\sqrt{3\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(2\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}}\; t^{\left[\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}. \tag{4.3}$$

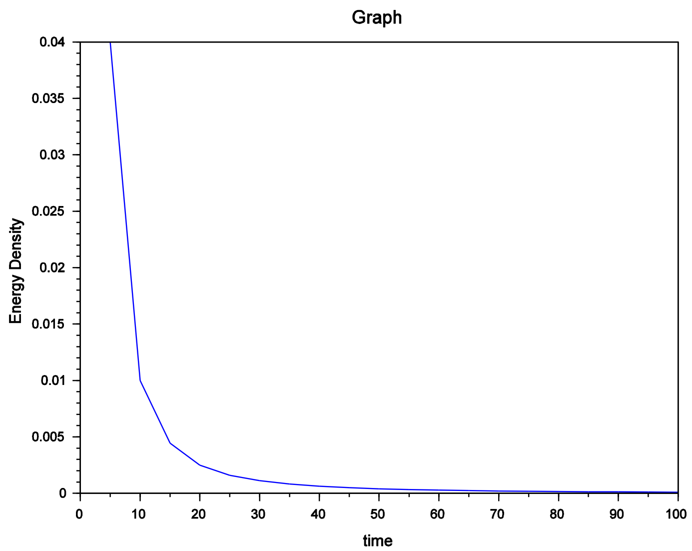

Figure 1: Energy density with time.

As illustrated in Figure 1, for $3f - 2\alpha^2 - 2nf > 0$ and $\alpha^2 < f$, the energy density decreases with cosmic time as $t \to \infty$, indicating that the Universe becomes progressively less dense due to its continuous expansion.

Pressure P:

Now calculate P using Eq. (3.31),

$P = \gamma\rho,$

putting $\gamma = 0,$

$$P = 0, \tag{4.4}$$

Therefore, the pressure tends to zero as cosmic time increases, suggesting that the matter content behaves like pressureless dust in the late-time evolution of the Universe.

Average Scale Factor (Hubble Parameter):

from the Eq. (3.35),

$$H = \left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \tag{4.5}$$

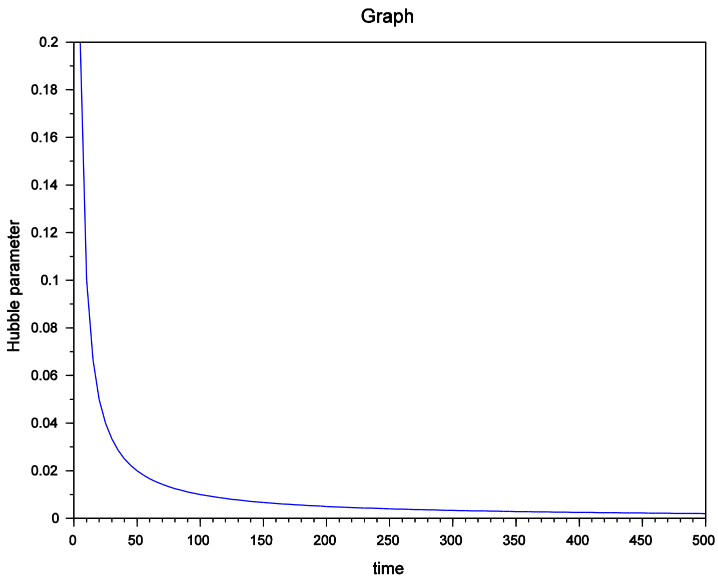

Figure 2: Hubble Parameter with time.

Therefore, for $3f - 2\alpha^2 - 2nf > 0$, the Hubble parameter tends to zero as t is large, indicating that the rate of cosmic expansion gradually decreases and approaches a negligible value at late cosmic times.

Expansion Scalar:

from the Eq. (3.37),

$$\theta = \left[\frac{6f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \tag{4.6}$$

Therefore, the expansion scalar tends to zero as t is large, implying that the cosmic expansion gradually slows down during the late-time evolution of the Universe.

Anisotropic Parameter:

from the Eq. (3.38),

$$A = \frac{1}{12}\, X^2\, {A_8}^2\; \frac{\left(f-2\alpha^2-2nf\right)^2}{f^2}\, t^{\left[\frac{(2f-4\alpha^2-4nf)}{(3f-2\alpha^2-2nf)}\right]}. \tag{4.7}$$

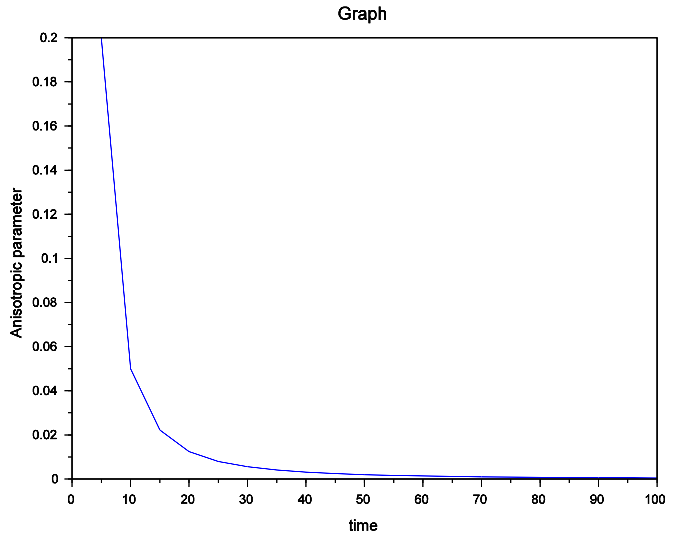

Figure 3: Anisotropic Parameter with time.

Therefore, assuming that $3f - 2\alpha^2 - 2nf > 0 \; and \; \alpha^2 < f$, the anisotropy parameter tends to zero as $t \to \infty$, it means that the directional differences in the expansion of the Universe gradually disappear.

Shear Scalar:

from the Eq. (3.39),

$$\sigma^2 = \frac{1}{2} X^2 \, {A_8}^2 \; \frac{(f-2\alpha^2-2nf)^2}{(3f-2\alpha^2-2nf)^2} \, t^{\left[\frac{-4f}{(2f-2\alpha^2-2nf)}\right]} . \tag{4.8}$$

here

$$A_8 = \frac{(3f-2\alpha^2-2nf)}{(f-2\alpha^2-2nf)} \left[\frac{\sqrt{3\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(2\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{-2f}{(3f-2\alpha^2-2nf)}} .$$

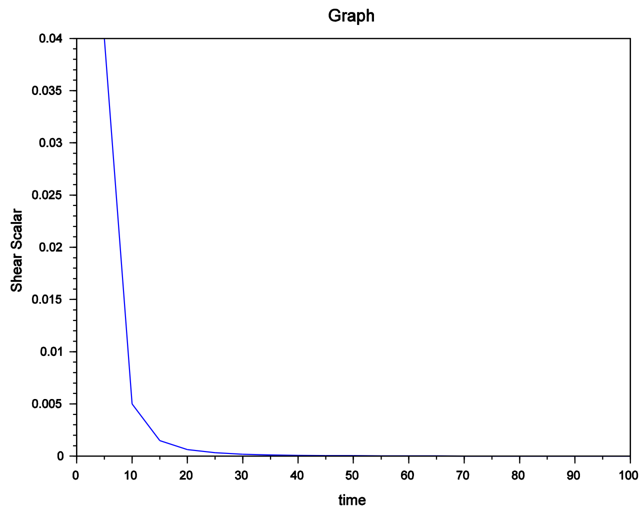

Figure 4: Shear Scalar with time.

Therefore, for $3f - 2\alpha^2 - 2nf > 0 \; and \; \alpha^2 < f$, the shear scalar tends to zero as $t \to \infty$, implying that the anisotropic deformation of the Universe gradually disappears.

The deceleration parameter q:

from the Eq. (3.40),

$$q = \frac{1}{2} - \frac{(\alpha^2+nf)}{f} = -ve \, , \tag{4.9}$$

The deceleration parameter is negative; the present cosmological model describes an accelerating Universe undergoing accelerated expansion. Thus, the present cosmological model exhibits accelerated expansion.

Case II: For $\gamma = \frac{1}{3}$ describes a radiation-dominated (non-relativistic) fluid:

Spatial Volume V:

Now calculate $g(V)$ using Eq. (3.28),

$$g(V) = A_1 \left[\frac{\sqrt{12\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(3\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}} t^{\left[\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}. \tag{4.10}$$

Now calculate C using Eq. (3.29),

$$C = \frac{A_1(3f-2\alpha^2-2nf)}{f(1-2n)} \left[\frac{\sqrt{12\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(3\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}} t^{\left[\frac{f(1-2n)}{(3f-2\alpha^2-2nf)}\right]}.$$

$$V = \left[\frac{\sqrt{12\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(3\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{2(\alpha^2-f)}{(3f-2\alpha^2-2nf)}} t^{\left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]}, \tag{4.11}$$

Therefore, assuming that $3f - 2\alpha^2 - 2nf > 0$ *and* $\alpha^2 < f$, the spatial volume tends to infinity at late times $t \to \infty$, demonstrating that the Universe undergoes continuous expansion throughout its evolution.

Energy density ρ:

Now calculate ρ using Eq. (3.30),

$$\rho = \frac{3\alpha^2 A_1{}^2 f}{2(3\alpha^2-f)} \left[\frac{\sqrt{12\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(3\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}} t^{\left[\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}. \tag{4.12}$$

Hence, for $3f - 2\alpha^2 - 2nf > 0$ *and* $\alpha^2 < f$, the energy density approaches zero as cosmic time becomes large, indicating the gradual dilution of the cosmic fluid during the evolution of the Universe.

Pressure P:

Now calculate P using Eq. (3.31),

$$P = \frac{\alpha^2 A_1{}^2 f}{2(3\alpha^2-f)} \left[\frac{\sqrt{12\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(3\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}} t^{\left[\frac{4(\alpha^2-f)}{(3f-2\alpha^2-2nf)}\right]}, \tag{4.13}$$

Therefore, assuming that $3f - 2\alpha^2 - 2nf > 0$ *and* $\alpha^2 < f$, the pressure tends to zero as *t* is large, suggesting that the cosmic fluid becomes effectively pressureless during the late-time evolution of the present cosmological model.

Average Scale Factor (Hubble Parameter):

from the Eq. (3.35),

$$H = \left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \tag{4.14}$$

Therefore, the Hubble Parameter tends to zero at *t* is large, indicating that the rate of cosmic expansion gradually decreases and approaches a negligible value at late cosmic times.

Expansion Scalar:

from the Eq. (3.37),

$$\theta = \left[\frac{6f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \tag{4.15}$$

Therefore, the expansion scalar tends to zero as *t* is large, implying that the cosmic expansion gradually slows down during the late-time evolution of the Universe.

Anisotropic Parameter:

from the Eq. (3.38),

$$A = \frac{1}{12} X^2 A_9{}^2 \; \frac{(f-2\alpha^2-2nf)^2}{f^2} t^{\left[\frac{(2f-4\alpha^2-4nf)}{(3f-2\alpha^2-2nf)}\right]}. \tag{4.16}$$

here

$$A_9 = \frac{(3f-2\alpha^2-2nf)}{(f-2\alpha^2-2nf)}\left[\frac{\sqrt{12\pi}\,\alpha A_1(3f-2\alpha^2-2nf)}{(3\alpha^2-f)^{\frac{1}{2}}\,(2\alpha^2+2nf-f)^{\frac{1}{2}}}\right]^{\frac{-2f}{(3f-2\alpha^2-2nf)}}.$$

Therefore, assuming that $3f-2\alpha^2-2nf>0\ and\ \alpha^2<f$, the anisotropy parameter tends to zero as t is large, implying that the Universe does not approach an isotropic state at late cosmic times.

Shear Scalar:

from the Eq. (3.39),

$$\sigma^2 = \frac{1}{2}X^2\,A_9{}^2\,\frac{(f-2\alpha^2-2nf)^2}{(3f-2\alpha^2-2nf)^2}\,t^{\left[\frac{-4f}{(2f-2\alpha^2-2nf)}\right]}. \qquad (4.17)$$

Therefore, for $3f-2\alpha^2-2nf>0\ and\ \alpha^2<f$, the shear scalar diverges as t is large, implying that the anisotropic effects become increasingly significant at late cosmic times.

The deceleration parameter q:

from the Eq. (3.40),

$$q = \frac{1}{2} - \frac{(\alpha^2+nf)}{f}, \qquad (4.18)$$

The deceleration parameter is negative; the present cosmological model describes an accelerating Universe undergoing accelerated expansion. Thus, the present cosmological model exhibits accelerated expansion.

Case III: For $\gamma = 1$ corresponds to the Zeldovich universe or stiff matter:

Now calculate $g(V)$ using Eq. (3.28), C using Eq. (3.29), ρ using Eq. (3.30), and P using Eq. (3.31)

$$g(V) = 0\,, V = 0, C = 0, \rho = 0, P = 0, \qquad (4.19)$$

Hubble Parameter:

from the Eq. (3.35),

$$H = \left[\frac{2f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \qquad (4.20)$$

Therefore, assuming that $3f-2\alpha^2-2nf>0\ and\ \alpha^2<f$, the Hubble Parameter tends to zero at t is large, indicating that the rate of cosmic expansion gradually decreases and approaches a negligible value at late cosmic times.

Expansion Scalar:

from the Eq. (3.37),

$$\theta = \left[\frac{6f}{(3f-2\alpha^2-2nf)}\right]\frac{1}{t}. \qquad (4.21)$$

Therefore, for $3f-2\alpha^2-2nf>0\ and\ \alpha^2<f$, the expansion scalar vanishes asymptotically as t is large, indicating that the expansion of the Universe becomes negligible at late cosmic times.

Anisotropic Parameter:

from the Eq. (3.38),

$$A = 0\,. \qquad (4.22)$$

Hence, the anisotropy parameter approaches zero in t at t is large, implying that the present cosmological model evolves towards isotropy.

Shear Scalar:

from the Eq. (3.39),

$$\sigma^2 = 0\,. \qquad (4.23)$$

Hence, the shear scalar approach zero at t is large, implying that the present cosmological model evolves towards anisotropic state.

The deceleration parameter q:

from the Eq. (3.40),

$$q = \frac{1}{2} - \frac{(\alpha^2 + nf)}{f}, \tag{4.24}$$

The negative deceleration parameter confirms that the present cosmological model is consistent with the observed late-time accelerated expansion of the Universe.

5. Stability

The model stability is dependent upon the function $c_s^2 = \frac{dP}{d\rho}$. If the function c_s^2 is greater than zero then the model is stable otherwise the model is unstable. This result taken from Katore, Hatkar, and Baxi[20].

Case I: For $\gamma = 0$,

Evaluate $\frac{dP}{d\rho}$ from Eq. (3.43) and Eq. (3.44) obtained as:

$$\frac{dP}{d\rho} = 0, \tag{5.1}$$

here the function c_s^2 is zero, therefore the model is unstable.

Case II: For $\gamma = \frac{1}{3}$,

Evaluate $\frac{dP}{d\rho}$ from Eq. (3.52) and Eq. (3.53) yielded as:

$$\frac{dP}{d\rho} = \frac{1}{3}, \tag{5.2}$$

here the function c_s^2 is Positive, therefore the model is stable.

Case III: For $\gamma = 1$,

Evaluate $\frac{dP}{d\rho}$ from Eq. (3.59) attained as:

$$\frac{dP}{d\rho} = 0, \tag{5.3}$$

here the function c_s^2 is zero, therefore the model is unstable.

The stability analysis reveals that the radiation-dominated model is stable, whereas the dust and Zeldovich-dominated models are unstable under the adopted stability criterion.

6. Conclusion

In this work, we have investigated a Bianchi Type I cosmological model with a variable gravitational constant (G) in the framework of Hoyle–Narlikar creation field theory. Exact solutions of the modified Einstein field equations have been obtained, and the physical and dynamical properties of the model have been analysed for three different equations of state corresponding to a dust-dominated universe ($\gamma = 0$), a radiation-dominated universe ($\gamma = \frac{1}{3}$), and the Zel'dovich (stiff matter) universe ($\gamma = 1$).

The obtained solutions show that the spatial volume increases without bound as cosmic time progresses, demonstrating that the Universe undergoes continuous expansion. The energy density decreases with time and approaches zero at late times, indicating that the cosmic fluid becomes increasingly dilute as the Universe expands. The Hubble parameter and the expansion scalar remain positive throughout the evolution but gradually decrease and asymptotically approach zero, implying that the expansion persists whilst its rate progressively diminishes.

For the dust-dominated universe ($\gamma = 0$), the pressure vanishes, consistent with the behaviour of pressureless matter. The anisotropy parameter and the shear scalar both approach zero as cosmic time increases, indicating that the anisotropic effects gradually disappear and the Universe evolves towards an isotropic state. The negative value of the deceleration parameter confirms that the model describes an accelerating Universe.

For the radiation-dominated universe ($\gamma = \frac{1}{3}$), the spatial volume continues to increase and the energy density decreases with time. The Hubble parameter and the expansion scalar decrease towards zero, whilst the deceleration parameter remains negative, indicating accelerated expansion. However, the behaviour of the anisotropy parameter and the shear scalar suggests that anisotropic effects persist at late times, implying that the model does not evolve towards isotropy in this case.

For the Zel'dovich (stiff matter) universe ($\gamma = 1$), the Hubble parameter, expansion scalar, anisotropy parameter, and shear scalar all tend to zero as cosmic time becomes large. Consequently, the model evolves towards a homogeneous and isotropic state whilst maintaining accelerated expansion, as indicated by the negative deceleration parameter.

The stability analysis reveals that the radiation-dominated model is stable under the adopted stability criterion, whereas the dust-dominated and Zel'dovich models are unstable. Overall, the proposed cosmological model exhibits continuous expansion, decreasing matter density, and accelerated cosmic evolution. Furthermore, the late-time isotropisation observed in the dust and Zel'dovich cases is consistent with the large-scale isotropy of the present Universe. These results demonstrate that the creation field framework with a variable gravitational constant provides a viable description of the dynamical evolution of anisotropic cosmological models and offers an interesting alternative approach to explaining the observed accelerated expansion of the Universe.

So it is clear that the universe is accelerating and isotropic at the time is large. The performance of the physical parameters is comparable to the obtained in the research paper Bali & Saraf [19]. The stability of the universe plays a major role in Creations theory. This is the novelty of its work. According to the cosmological explanations this cosmological model is unstable for $\gamma = 0, \gamma = 1,$ and stable for $\gamma = \frac{1}{3}$. The performance of the physical parameters is comparable to the obtained in the research paper Dagwal & Pawar [15]. The spatial volume of the model grows exponentially over time, representing an inflationary universe. The matter density remains positive, satisfying the reality condition. The model also adheres to the relation $A^2 = \frac{\sigma^2}{3H^2} = 0.01$, which is a very small value, as highlighted by Crawford and Aguiar [21]. The study focuses on the matter-dominated stage in creation field cosmology, emphasizing that the creation field exists at both the beginning and the end of the universe.

Acknowledgments

The authors are appreciative of the unknown arbitrator for his product ideas to improve the quality of the standard.

References

[1]. Riess, A. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. Garnavich, ... and J. Tonry, Observational evidence from supernovae for an accelerating universe and a cosmological constant, The Astronomical Journal, 116(3), 1009, (1998).

[2]. S. Perlmutter, G. Aldering, G. Goldhaber, R. Knop, P. Nugent, P. Castro, ... and Supernova Cosmology Project, Measurements of Omega and Λ from 42 high-redshift supernovae, The Astrophysical Journal, 517(2), 565, (1999).

[3]. C. Brans, and R. Dicke, Mach's principle and a relativistic theory of gravitation, Physical review, 124(3), 925, (1961).

[4]. D. Saez, and V. Ballester, A simple coupling with cosmological implications, Physics letters A, 113(9), 467--470, (1986).

[5]. F. Hoyle and J. Narlikar, On the avoidance of singularities in C-field cosmology, Proceedings of the Royal Society of London, Mathematical and Physical Sciences, 278, 465, 1964.

[6]. R. Dicke, and P. Peebles, Gravitation and space science, Space Science Reviews, 4(4), 419-460, (1965).

[7]. F. Hoyle, and J. Narlikar, A radical departure from the 'steady-state'concept in cosmology, Proceedings of the Royal Society of London Series A. Mathematical and Physical Sciences, 290(1421), 162-176, (1966).

[8]. J. Narlikar and T. Padmanabhan, Creation field cosmology: a possible solution to singularity, horizon and flatness problem, Physical Review D, 39, 1928, (1985).

[9]. P. Singh, M. Sami, and N. Dadhich, Cosmological dynamics of a phantom field Physical Review D, 68(2), 023522, (2003).

[10]. H. Giacomini, and L. Lara, Qualitative dynamical properties of a phantom field with an arbitrary potential, General Relativity and Gravitation, 38, 137-144, (2006).

[11]. R. Vishwakarma, and J. Narlikar, Modeling repulsive gravity with creation, Journal of Astrophysics and Astronomy, 28, 17-27, (2007).

[12]. R. Bali, and R. Tikekar, C-field Cosmology with Variable G in the Flat Friedmann–Robertson–Walker Model, Chinese Physics Letters, 24(11), 3290, (2007).

[13]. R. Bali, and M. Kumawat, C-field Cosmological Models with Variable G in FRW Space–Time, International Journal of Theoretical Physics, 48, 3410-3415, (2009).

[14]. T. Saraf, Bianchi type I cosmological model for dust distribution with variable G, JRAOPS, 16, 175-182, (2017).

[15]. V. Dagwal, and D. Pawar, Tilted Two Fluids Cosmological Models with Variable G and {Λ In General Relativity, arXiv preprint arXiv: 1704 07107, (2017).

[16]. A. Alfedeel, Bianchi type–I Model with Time Varying Λ and G, The Generalized Solution. Open Astronomy, 29(1), 89-93, (2020).

[17]. K. Singh, J. Baro, and A. Meitei, Higher Dimensional Bianchi Type-I Cosmological Models With Massive String in General Relativity, Frontiers in Astronomy and Space Sciences, 8, 777554, (2021).

[18]. R. Tiwari, A. Alfedeel, D. Sofuoğlu, A. Abebe, E. Hassan, and B. Shukla, A cosmological model with time dependent Λ, G and viscous fluid in general relativity, Frontiers in Astronomy and Space Sciences, 9, 965652, (2022).

[19]. R. Bali, and S. Saraf, Creation Field Cosmological Model with Variable Cosmological Term (Λ) in Bianchi Type III Space-Time, Advances in High Energy Physics, 2024(1), 5901224, (2024).

[20]. S. Katore, S. Hatkar, and R. Baxi, Domain wall cosmological models with deceleration parameter in modified theory of gravitation, Chinese Journal of Physics, 54(4), 563-573, (2016).

[21]. P. Crawford, and P. Aguiar, Cosmology 2000 in Conference Proc. NASA ADS, pp., Northern Ireland, 2000(1), 69–76, (2000).

Cite this Article:

Gupta, A.R. (2026). Stability Behavior of Bianchi Type I Cosmological Models with Variable G under a Creation Field. Pi International Journal of Mathematical Sciences, 2(4), 1–14.

Journal URL: *https://pijms.com/*

DOI: *https://doi.org/10.59828/pijms.v2i4.42*