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On the *S*-Menger like Space

Neeraj Malviya[1], Pradip Kumar Keer[2a, 2b], and Geeta Agrawal[3]
[1]Swami Vivekanand Government P.G. College (PMCOE), Raisen, Madhya Pradesh, India
[2a]Research Scholar, Government Motilal Vigyan Mahavidyalaya (M.V.M.), Bhopal, Madhya Pradesh, India
[2b]Chameli Devi Institute of Professional Studies, Indore, Madhya Pradesh, India
[3]Government Motilal Vigyan Mahavidyalaya (M.V.M.), Bhopal, Madhya Pradesh, India
[2]**Corresponding Author Email ID:** pradip.keer@gmail.com

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Abstract

In the present paper, we introduce the notion of S-Menger like space (S-MLS), which is the hybrid structure of metric like space and S-Menger space. We also given examples of (S-MLS). In the last section we prove Banach contraction principle and another unique fixed-point theorem for Menger k contractive map in the setting of S-Menger like Space.

Keywords: *S-Menger like space, Fixed point, Banach contraction principle.*
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1. Introduction

Karl Menger started the theory of probabilistic metric spaces in 1942 [13], introducing the idea of a probabilistic metric space by substituting a distribution function for the distance between two locations. This groundbreaking concept, which offered a mathematical framework for simulating uncertainty in distance, has grown to be a significant field of study in fixed-point theory, topology, and nonlinear analysis. Triangular norms, or t-norms, were subsequently introduced by Schweizer and Sklar [8] to further the theory of statistical metric spaces. These norms are essential to the creation of probabilistic metric spaces.

Fixed-point theory has been greatly enhanced by the creation of generalized metric spaces. The concepts of 2-metric spaces were proposed by Gahler in 1963 and 1966 [4,5], which sparked a great deal of research in generalized distance spaces. Dhage later established a number of fixed-point theorems in this context and introduced D-metric spaces [1]. D-metric spaces offered a significant generalization, although their topological structure was later found to have several flaws. G-metric spaces [12] were developed by Mustafa and Sims to address these issues, and they have since grown to be one of the most significant generalized metric structures in fixed-point theory. Sedghi et.al [7] defined D^* metric space which generalize the structure of G-metric space.

Sedghi, Shobe, and Aliouche proposed the concept of S-metric spaces in 2012 [6], motivated by these advancements. S-metric spaces maintain desirable topological qualities while unifying and generalizing a number of current metric structures. They have garnered a lot of interest since they were first introduced, and many fixed-point findings under different contractive situations have been established. Several authors, such as An and Dung [10], have also looked into a number of expansions and uses of S-metric spaces. On the other hand, fixed point theorems applicable in Economics [2].

Sarkar, Das, and Pramanik [9] made a major contribution in this direction by introducing the notion of S-Menger spaces by fusing the geometric framework of S-metric spaces with the probabilistic structure of Menger spaces. They proved that this framework naturally extends both Menger spaces and S-metric spaces, and they produced fixed-point theorems of the Banach and Kannan types in S-Menger spaces. Keer et.al [3] defined asymptotically regular maps in S-Menger space [3].

Motivated by the above developments, the objective of the present paper is to introduce a new generalized probabilistic structure, called the S-Menger-like space. We investigate its fundamental properties, and provide illustrative examples to demonstrate the validity of the proposed concept and its relationship with existing generalized probabilistic metric spaces.

In the last section the famous Banach contraction theorem and fixed-point theorem for Menger k contractive maps are established.

2. Preliminaries

Definition 2.1.[8] A map $*: [0,1]^3 \rightarrow [0,1]$ is a continuous t-norm if it meets the requirements listed below:

T_1:$* (\xi, 1,1) = \xi, * (0,0,0) = 0$

T_2:$* (\xi, \varsigma, \eta) = * (\xi, \eta, \varsigma) = * (\varsigma, \eta, \xi)$

T_3:$* (\xi_1, \varsigma_1, \eta_1) \geq * (\xi_2, \varsigma_2, \eta_2,)$ for $\xi_1 \geq \xi_2, \varsigma_1 \geq \varsigma_2, \eta_1 \geq \eta_2$

examples of t-norm are

(1): $*_p (\xi, \varsigma, \eta) = \xi . \varsigma . \eta$ product t-norm;

(2): $*_m (\xi, \varsigma, \eta) = min\{\xi, \varsigma, \eta\}$ minimum t-norm

3. Main Results

Definition 3.1. S-Menger like space (Dislocated S-Menger Space).

S-Menger like space is defined as the 3-tuple $(X, S_l, *)$ if X is a non-empty set, S_l is a function defined on X^3 to the set of distribution function and * is a continuous third order t-norm such that the following conditions are satisfied:

(i) $S_{l(\xi,\varsigma,\eta)}(0) = 0$ for all $\xi, \varsigma, \eta \in X$

(ii) $S_{l(\xi,\xi,\eta)}(t) < 1$ for $t > 0$ with $\xi \neq \varsigma$,

(iii) $S_{l(\xi,\varsigma,\eta)}(t) = 1$ for all $t > 0$, then $\xi = \eta = \varsigma$.

(iv) $S_{l(\xi,\varsigma,\eta)}(t) \geq \{S_{l(\xi,\xi,p)}(t_1) * S_{l(\varsigma,\varsigma,p)}(t_2) * S_{l(\eta,\eta,p)}(t_3)\}$,

Where $t = t_1 + t_2 + t_3$ and $t, t_1, t_2, t_3 > 0$ for all $\xi, \varsigma, \eta, p \in X$

Example 3.2. Let $X = R$ be a real line and * be the product T norm.

and let S_l be the distribution function on X^3 defined by

$$S_{l(\xi,\varsigma,\eta)}(t) = \begin{cases} 0 & ,t = 0 \\ exp^{\left[\frac{|\xi+\eta|+|\varsigma+\eta|}{t}\right]^{-1}} & ,t > 0. \end{cases}$$

for all $\xi, \varsigma, \eta \in X$ *and* t > 0.

Then $(R, S_l,*)$ is an S-Menger like space (Dislocated S-Menger Space).

Example 3.3. Let $X = R^+$ be a real line and * be the product T norm and let S_l be the distribution function on X^3 defined by

$$S_{l(\xi,\varsigma,\eta)}(t) = \begin{cases} 0 & ,t = 0 \\ exp^{\left[\frac{\max\{\xi,\eta\}+\max\{\varsigma,\eta\}}{t}\right]^{-1}} & ,t > 0. \end{cases}$$

for all $\xi, \varsigma, \eta \in X$ *and* t > 0.

Then $(R, S_l,*)$ is an S-Menger like space(Dislocated S-Menger Space) .

Remark 1: It is quite obvious to prove (i) , (ii), (iii) and (iv) of definition 3.1.

Remark 2: In above example $\xi = \eta = \varsigma$ *need not imply* $S_{l(\xi,\varsigma,\eta)}(t) = 1$. So it is different structure rather than S-Menger Space.

Every S-Menger space is a S-Menger - like space but every S-Menger like space need not be S-Menger space since $S_{l(\xi,\xi,\xi)}(t)$ need not be 1 , $\forall \xi \in X$

The following example shows that S-Menger like space (Dislocated S-Menger Space) need not be symmetric i.e. $S_{l(\xi,\xi,\varsigma)}(t) \neq S_{(\varsigma,\varsigma,\xi)}(t)$ but every S-Menger space is Symmetric.

Example 3.4. let $X = R$ real line and * be the product T norm and let S_l be the distribution function on X^3 defined by

$$S_{l(\xi,\varsigma,\eta)}(t) = \begin{cases} 0 & ,t = 0 \\ \dfrac{t}{t + |\xi - \eta| + |\varsigma - \eta| + |\xi| + |\varsigma| + |\eta|} & ,t > 0 \end{cases}$$

for all $\xi, \varsigma, \eta \in X$.Then $(R, S_l,*)$ is an S-Menger like space (Dislocated S-Menger Space) and $S_{l(\xi,\xi,\varsigma)}(t) \neq S_{l(\varsigma,\varsigma,\xi)}(t)$ for $\xi \neq \varsigma$.

Definition 3.5. A sequence $\{\xi_n\}$ in $(X, S_l,*)$ is converges to $\xi \in X$ if

$S_{l(\xi_n,\xi_n,\xi)}(t) \to S_{l(\xi,\xi,\xi)}(t)$ as $n \to \infty$ for each $t > 0$.

Definition 3.6. Let $(X, S_l,*)$ be an S-Menger like space and $\{\xi_n\}$ be a sequence in X is said to be Cauchy sequence, if $\lim_{n\to\infty} S_{l(\xi_{n+p},\xi_{n+p},\xi)}(t)$ for all $t > 0, p \geq 1$ exists and is finite .

Definition 3.7. A S-Menger like space $(X, S_l,*)$ is said to be complete if all of its Cauchy sequences $\{\xi_n\}$ in X converges to some $\xi \in X$ such that

$$\lim_{n\to\infty} S_{l(\xi_n,\xi_n,\xi)}(t) = S_{l(\xi,\xi,\xi)}(t) = \lim_{n\to\infty} S_{l(\xi_{n+p},\xi_{n+p},\xi)}(t) \text{ for all } t > 0, p \geq 1$$

Definition 3.8. The S −Menger like space is called symmetric if $S_{(\xi,\xi,\varsigma)}(t) = S_{(\varsigma,\varsigma,\xi)}(t)$.

Definition 3.9. Let $(X, S_l, *)$ be a S-Menger like spaces. A mapping $T: X \to X$ is said to be Menger contractive if there exists $k \in (0,1)$ such that

$$\frac{1}{S_{l(T\xi,T\varsigma,T\eta)}(t)} - 1 \leq k[\frac{1}{S_{l(\xi,\varsigma,\eta)}(t)} - 1]$$

for all $\xi, \varsigma, \eta \in X$ and $t > 0$. Here k is called the Menger contractive constant of T.

4. Fixed Point Theorems

Theorem 4.1. Let $(X, S_l, *)$ be a complete symmetric S-Menger like space such that

$\lim_{n\to\infty} S_{l(\xi,\varsigma,\eta)}(t) = 1$, for all $\xi, \varsigma, \eta \in X, t > 0$ and $T: X \to X$ be a mapping satisfying the condition

$S_{l(T\xi,T\varsigma,T\eta)}(at) \geq S_{l(\xi,\varsigma,\eta)}(t)$ .... (4.1.1)

for all $\xi, \varsigma, \eta \in X, t > 0$, where $a \in (0,1)$. Then T has a unique fixed point $w \in X$ and $S_{l(w,w,w)}(t) = 1$, for all $t > 0$.

Proof. Let $(X, S_l, *)$ be a complete symmetric S-Menger like space. For a given element $\xi_0 \in X$, define a sequence $\{\xi_n\}$ in X by

$\xi_1 = T\xi_0, \xi_2 = T^2\xi_0 = T\xi_1 \dots\dots, \xi_n = T^n\xi_0 = T\xi_{n-1}$ for all $n \in \mathbb{N}$

If $\xi_n = \xi_{n-1}$ for some $n \in \mathbb{N}$,then ξ_n is a fixed point of T. We assume that $\xi_n \neq \xi_{n-1}$ for all $n \in \mathbb{N}$. For all $t > 0$ and $n \in \mathbb{N}$, we get from (4.1.1) that

$$S_{l(\xi_n,\xi_n,\xi_{n+1})}(t) \geq S_{l(\xi_{n+1},\xi_{n+1},\xi_n)}(at) = S_{l(T\xi_n,T\xi_n,T\xi_{n-1})}(at) \geq S_{l(\xi_n,\xi_n,\xi_{n-1})}(t)$$

for all $n \in \mathbb{N}$ and $t > 0$. Therefore, by applying the above expression, we can deduce that

$S_{l(\xi_{n+1},\xi_{n+1},\xi_n)}(t) \geq S_{l(\xi_{n+1},\xi_{n+1},\xi_n)}(at) = S_{l(T\xi_n,T\xi_n,T\xi_{n-1})}(at) \geq S_{l(\xi_n,\xi_n,\xi_{n-1})}(t)$....... (4.1.2)

$$= S_{l(T\xi_{n-1},T\xi_{n-1},T\xi_{n-2})}(t) \geq S_{l(\xi_{n-1},\xi_{n-1},\xi_{n-2})}\left(\frac{t}{a}\right) \geq \cdots \geq S_{l(\xi_1,\xi_1,\xi_0)}\left(\frac{t}{a^n}\right)$$

for all $n \in \mathbb{N}, p \geq 1$ and $t > 0$. Thus, we have

$$S_{l(\xi_n,\xi_n,\xi_{n+p})}(t) \geq \{S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_{n+p},\xi_{n+p},\xi_{n+1})}\left(\frac{t}{3}\right)\}$$

Continuing in this way, we get

$$S_{l(\xi_n,\xi_n,\xi_{n+p})}(t) \geq \{S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_{n+1},\xi_{n+1},\xi_{n+2})}\left(\frac{t}{3^2}\right) * S_{l(\xi_{n+1},\xi_{n+1},\xi_{n+2})}\left(\frac{t}{3^2}\right) * \dots\dots.$$

$$* S_{l(\xi_{n+p-1},\xi_{n+p-1},\xi_{n+p})}\left(\frac{t}{3p}\right)\}.$$

Using (4.1.2) in the above inequality, we have

$S_{l(\xi_n,\xi_n,\xi_{n+p})}(t) \geq \{S_{l(\xi_0,\xi_0,\xi_1)}\left(\frac{t}{3a^n}\right) * S_{l(\xi_0,\xi_0,\xi_1)}\left(\frac{t}{3a^{n+1}}\right) *, \dots .* S_{l(\xi_0,\xi_0,\xi_1)}\left(\frac{t}{3^p a^{n+p-1}}\right)\}$ (4.1.3)

We know that $\lim_{n\to\infty} S_{l(\xi,\varsigma,\eta)}(t) = 1$, for all $\xi, \varsigma, \eta \in X, t > 0$, where $a \in (0,1)$. It follows from (4.1.3) that

$\lim_{n\to\infty} S_{l(\xi_n,\xi_n,\xi_{n+p})}(t) = 1 * 1 *, \dots ., * 1 = 1$ for all $t > 0 , p \geq 1$

Hence $\{\xi_n\}$ is a Cauchy sequence.

The completeness of $(X, S_l,*)$ be a complete symmetric S-Menger like space ensures that there exists $w \in X$ such that

$\lim_{n\to\infty} S_{l(\xi_n,\xi_n,w)}(t) = \lim_{n\to\infty} S_{l(\xi_n,\xi_n,\xi_{n+p})}(t) = S_{l(w,w,w)}(t) = 1,$ for all $t > 0, p \geq 1$ ... (4.1.4)

Now, we show that $w \in X$ is a fixed point of T. We have

$$S_{l(w,w,Tw)}(t) \geq \{ S_{l(w,w,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(w,w,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(Tw,Tw,\xi_{n+1})}\left(\frac{t}{3}\right)\}$$

for all $t > 0$. Since $\xi_{n+1} = T\xi_n$

$$\Rightarrow S_{l(w,w,Tw)}(t) \geq \{ S_{l(w,w,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(w,w,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(Tw,Tw,T\xi_n)}\left(\frac{t}{3}\right)\}$$

$$S_{l(w,w,Tw)}(t) \geq \{ S_{l(w,w,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(w,w,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(w,w,\xi_n)}\left(\frac{t}{3a}\right)\}$$

Taking the limit as $n \to \infty$, and by (4.1.4),

$$\lim_{n\to\infty} S_{l(w,w,\xi_{n+1})}\left(\frac{t}{3}\right) = 1, and \ \lim_{n\to\infty} S_{l(w,w,\xi_n)}\left(\frac{t}{3a}\right) = 1.$$

Hence $$S_{l(w,w,Tw)}(t) \geq \{1 * 1 * 1\} = 1.$$

Therefore, w is a fixed point of T and $S_{l(w,w,w)}(t) = 1$, for all , $t > 0$. Now, we investigate the uniqueness of fixed point. For this, assume that v and w are two fixed point of T. Then by (4.1.1), we have

$$S_{l(w,w,v)}(t) = S_{l(Tw,Tw,Tv)}(t) \geq S_{l(w,w,v)}\left(\frac{t}{a}\right)$$

for all $t > 0$. Thus, we obtain

$S_{l(w,w,v)}(t) \geq S_{l(w,w,v)}\left(\frac{t}{a^n}\right)$ for all $n \in \mathbb{N}$

Taking the limit as $n \to \infty$ and using $\lim_{n\to\infty} S_{l(w,w,v)}(t) = 1$,we get $w = v$.

Hence the fixed point is unique.

Theorem 4.2. Let $(X, S_l,*)$ be a complete symmetric S-Menger like spaces and $T: X \to X$, be a contractive mapping with Menger contractive constant k , then T has a unique fixed point $u \in X$ and $S_{l(u,u,u)}(t) = 1$ for all $t > 0$.

Proof. Let $(X, S_l,*)$ be a complete symmetric S-Menger like spaces. For a given element $\xi_0 \in$ X, define a sequence $\{\xi_n\}$ in X by

$$\xi_1 = T\xi_0, \xi_2 = T^2\xi_0 = T\xi_1 \ldots \ldots, \xi_n = T^n\xi_0 = T\xi_{n-1} \text{ for all } n \in \mathbb{N}$$

If $\xi_n = \xi_{n-1}$ for some $n \in \mathbb{N}$ then ξ_n is a fixed point of T.

Therefore, we assume that $\xi_n \neq \xi_{n-1}$ for all $n \in \mathbb{N}$. For any $n \in \mathbb{N}$ and $t > 0$, we obtain from definition (3.9) that

$$\frac{1}{S_{l(\xi_n,\xi_n,\xi_{n+1})}(t)} - 1 = \frac{1}{S_{l(T\xi_{n-1},T\xi_{n-1},T\xi_n)}(t)} - 1 \leq k\left[\frac{1}{S_{l(\xi_{n-1},\xi_{n-1},\xi_n)}(t)} - 1\right].$$

Now setting $S_{l(\xi_n,\xi_n,\xi_{n+1})}(t) = S_{l(n)}(t)$ and $1 - k = \lambda$, it follows from the above inequality that

$$\frac{1}{S_{l(n)}(t)} \leq \frac{k}{S_{l(n-1)}(t)} + \lambda \text{ for all } t > 0.$$

From successive applications of the above inequality, we obtain

$$\frac{1}{S_{l(n)}(t)} \leq \frac{k^n}{S_{l(0)}(t)} + k^{n-1}\lambda + k^{n-2}\lambda + \cdots..+\lambda$$

$$\leq \frac{k^n}{S_{l(0)}(t)} + (k^{n-1} + k^{n-2} + \cdots..+1)\lambda$$

$$\leq \frac{k^n}{S_{l(0)}(t)} + (1-k^n).$$

thus, $\frac{1}{\frac{k^n}{S_{l(0)}(t)}+(1-k^n)} \leq S_{l(n)}(t)$ for all $t > 0$ $n \in \mathbb{N}$. (4.2.1)

If $n \in \mathbb{N}, p \geq 1$, then we have

$$S_{l(\xi_n,\xi_n,\xi_{n+p})}(t) \geq \{S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_{n+p},\xi_{n+p},\xi_{n+1})}\left(\frac{t}{3}\right)\}$$

$$\geq \{S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_n,\xi_n,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(\xi_{n+1},\xi_{n+1},\xi_{n+2})}\left(\frac{t}{3^2}\right) * \ldots\ldots * S_{l(\xi_{n+p-1},\xi_{n+p-1},\xi_{n+p})}\left(\frac{t}{3^p}\right)\}.$$

using the above inequality, we have

$S_{l(\xi_n,\xi_n,\xi_{n+p})}(t)$

$$\geq \{\frac{1}{\frac{k^n}{S_{l(0)}\left(\frac{t}{3}\right)}+(1-k^n)} * \frac{1}{\frac{k^n}{S_{l(0)}\left(\frac{t}{3}\right)}+(1-k^n)} * \frac{1}{\frac{k^{n+1}}{S_{l(0)}\left(\frac{t}{3^2}\right)}+(1-k^{n+1})} * \ldots.$$

$$* \frac{1}{\frac{k^{n+p-1}}{S_{l(0)}\left(\frac{t}{3^{p-1}}\right)}+(1-k^{n+p-1})}\}$$

$$\geq \{\frac{1}{\frac{k^n}{S_{l(0)}\left(\frac{t}{3}\right)}+1} * \frac{1}{\frac{k^{n+1}}{S_{l(0)}\left(\frac{t}{3}\right)}+1} * \ldots.* \frac{1}{\frac{k^{n+p-1}}{S_{l(0)}\left(\frac{t}{3^{p-1}}\right)}+1}\}$$

As $k \in (0,1)$, therefore using the properties of continuous *t*-norm we obtain from the above inequality that $\lim_{n\to\infty} S_{l(\xi_n\,\xi_n,\xi_{n+p})}(t) = 1$, for all $t > 0$, $p \geq 1$.

Therefore $\{\xi_n\}$ is a Cauchy sequence in $(X, S_l,*)$.

The completeness of $(X, S_l,*)$ ensures that there exists $u \in X$ such that

$\lim_{n\to\infty} S_{l(\xi_n,\xi_n,u)}(t) = \lim_{n\to\infty} S_{l(\xi_n,\xi_n,\xi_{n+p})}(t) = S_{l(u,u,u)}(t) = 1,$ for all $t > 0$, $p \geq 1$ ... (4.2.2)

Now we show that u is a fixed point of T.

Again, for all $t > 0$, $n \geq 0$, we obtain from (1) that

$$\frac{1}{S_{l(T\xi_n,T\xi_n,Tu)}(t)} - 1 \leq k\left[\frac{1}{S_{l(\xi_n,\xi_n,u)}(t)} - 1\right] = \frac{k}{S_{l(\xi_n,\xi_n,u)}(t)} - k,$$

that is,

$$\frac{1}{\frac{k}{S_{l(\xi_n,\xi_n,u)}(t)}+(1-k)} \leq S_{l(T\xi_n,T\xi_n,Tu)}(t).$$

Using the above inequality, we obtain

$$S_{l(u,u,Tu)}(t) \geq \{S_{l(u,u,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(u,u,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(Tu,Tu,\xi_{n+1})}\left(\frac{t}{3}\right)\}$$

$$= \{S_{l(u,u,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(u,u,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(Tu,Tu,T\xi_n)}\left(\frac{t}{3}\right)\}$$

$$\geq \{S_{l(u,u,\xi_{n+1})}\left(\frac{t}{3}\right) * S_{l(u,u,\xi_{n+1})}\left(\frac{t}{3}\right) * \frac{1}{\frac{k}{S_{l(\xi_n,\xi_n,u)}\left(\frac{t}{3}\right)}+1-k}.$$

Taking limit as $n \to \infty$ and using (4.2.2) in the above inequality we obtain that $S_{l(u,u,Tu)}(t) = 1$, that is, $Tu = u$. Therefore, u is a fixed point of T and $S_{l(u,u,u)}(t) = 1$ for all $t > 0$.

For uniqueness, let v be another fixed point of all T. Suppose, $S_{l(u,u,v)}(t) < 1$ for some $t > 0$, then it follows from definition (3.9) that

$$\frac{1}{S_{l(u,u,v)}(t)} - 1 = \frac{1}{S_{l(Tu,Tu,Tv)}(t)} - 1$$

$$\leq k\left[\frac{1}{S_{l(u,u,v)}(t)}-1\right]<\frac{1}{S_{l(u,u,v)}(t)}-1,$$

which is a contradiction. Therefore, we must have $S_{l(u,u,v)}(t)=1$, for all $t>0$, and therefore $u=v$.

Corollary 4.3. Let $(X, S_l,*)$ be a complete symmetric *S*-Menger like spaces and $T: X\to X$, be a mapping

$\frac{1}{S_{l(T^n\xi,T^n\varsigma,T^n\eta)}(t)}-1\leq k[\frac{1}{S_{l(\xi,\varsigma,\eta)}(t)}-1]$

for some positive integer n, and for all $\xi,\varsigma,\eta\in X, t>0$, where $k\in(0,1)$. Then T has a unique fixed point $u\in X$ and $S_{l(u,u,v)}(t)=1$ for all $t>0$.

Proof. According to theorem (4.2), $S_{l(u,u,v)}(t)=1$ for every $t>0$, and T^n has a unique fixed-point u. Since $T^n(Tu)=T(T^n u)=Tu$,, Tu is thus a fixed point of T^n, and since $Tu=u$ is unique. The fixed point of *T* is unique since it is also a fixed point of T^n.

Example 4.4. Let $X=[0,1]$. Define * by $a*b*c=a.b.c$ and distribution function on X^3 by

$$S_{l(\xi,\varsigma,\eta)}(t)=\begin{cases}0, & t=0\\ \dfrac{t}{t+\max\{\xi,\eta\}+\max\{\varsigma,\eta\}}, & t>0\end{cases}$$

for $\xi,\varsigma,\eta\in X, t>0$.

Then $S_{l(\xi,\varsigma,\eta)}(t)$ be a complete symmetric *S*-Menger like space.

If $T: X\to X$ is given by $T(\xi)=\begin{cases}0 & \text{if } \xi=0\\ \frac{\xi}{2} & \text{, otherwise}\end{cases}$

The Menger contractive constant $k\in\left[\frac{1}{2},1\right)$ indicates that T is a Menger contractive mapping. As a result, theorem (4.2) requirements are all met. Additionally, $S_{l(0,0,0)}(t)=1$ for all $t>0$, and *T* has a unique fixed point $0\in X$.

Conclusion

The notion of *S*-Menger-like spaces not only extends the framework of *S*-Menger spaces and like Menger space but also provides a suitable foundation for future investigations in fixed-point theory. The fixed-point theorems presented in this paper may have great applications and generalizations in pure and applied sciences.

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