Pi International Journal of Mathematical Sciences
Volume 2 | Issue 4 | August 2026 | ISSN: 3107-9830 (Online)
Available online at: https://pijms.com/
DOI: https://doi.org/10.59828/pijms.v2i4.44

Centered Octagonal Sum Labeling of Some Families of Graphs

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Received: 19 July 2026 | **Accepted:** 15 August 2026 | **Published:** 30 August 2026

Abstract

A graph G with "p" vertices and "q" edges is called a Centered Octagonal sum graph(COSG) if it admits a labeling known as Centered Octagonal sum labeling(COSL). COSL is an injective function $h: V(G) \longrightarrow N$, *where N represents the set of all non- negative integers that induces a bijection* $h^+ : E(G) \to \{ cP_8(1), cP_8(2), \cdots, cP_8(q) \}$ *of the edges of G defined by* $h^+(uv) = h(u) + h(v)$ *for every* $e = uv \in E(G)$, *where* $cP_8(1), cP_8(2), \cdots, cP_8(q)$ *are the first q Centered Octagonal numbers. In this work, we prove that various families of graphs, such as Caterpillars, Olive trees, Shrub* $St(n_1, n_2, \cdots, n_m)$, *Banana tree* $Bt(n_1, n_2, \cdots, n_m)$, $H - graphs$ *and* $H \odot K_1$ *are Centered Octagonal sum graphs.*

Keywords: *Centered Octagonal numbers, Centered Octagonal sum labeling and Centered Octagonal sum graphs.*
Mathematics Subject Classification (MSC) 2020*: 05C78.*

Introduction

In this paper, we focus on non-trivial finite, simple undirected graphs. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. We adopt the graph theoretic notations and terminology from Bondy and Murty [2] and the number theory concepts from Burton[1]. Numerous types of labeling' s have been introduced and explored by various researchers, and an excellent review of graph labeling can be found in[4].

Definition 1.1

Numbers of the form $cP_8 = (2n-1)^2, for\ all\ n \geq 1$ are called Centered Octagonal numbers. The first few Centered Octagonal numbers are $1, 9, 25, 49, 81, \cdots$

Definition 1.2

COSL is an injective function $h: V(G) \longrightarrow N$, where N represents the set of all non- negative integers that induces a bijection $h^+ : E(G) \to \{ cP_8(1), cP_8(2), \cdots, cP_8(q) \}$ of the edges of G defined by $h^+(uv) = h(u) + h(v)$ for every $e = uv \in E(G)$. where $cP_8(1), cP_8(2), \cdots, cP_8(q)$ are the first q Centered Octagonal numbers

Definition 1.3

A graph G is called a COSG if it admits COSL.

Definition 1.4

Shrub $St(n_1, n_2, n_3, \ldots, n_m)$ is a graph obtained by connecting a vertex v_0 to the central vertex of each of m numbers of stars

Definition 1.5

Banana tree denoted by $Bt(n_1, n_2, n_3, \dots, n_m)(m\ times\ n)$ is a graph obtained by connecting a vertex v_0 to one leaf of each of m number of stars

2.Main Results

Theorem 2.1

Caterpillars $S(n_1, n_2, n_3, \dots, n_m)$ are COSG for all $n_r \geq 1$

Proof

Let $P_m: u_1 u_2 u_3 \cdots u_m$ be a path with m vertices. From each vertex u_s , $s = 1,2, \dots, m$ there are n_r , $r = 1,2,3, \dots, m$ pendent vertices say $u_{s1}, u_{s2}, u_{s3}, \dots, u_{sn_r}$. The resulting graph is known as caterpillar and is represented as $S(n_1, n_2, n_3, \dots, n_m)$.

The caterpillar graph $S(n_1, n_2, n_3, \dots, n_m) = G$ has $m + n_1 + n_2 + n_3 + \cdots + n_m$ vertices and $q = m - 1 + n_1 + n_2 + n_3 + \cdots + n_m$ edges.

Define $h: V(G) \longrightarrow N$ by

$$h(u_1) = 0$$
$$h(u_s) = cP_8(s-1) - h(u_{s-1}) \ ; 2 \leq s \leq m$$
$$h(u_{1r}) = cP_8(m-1+r) \quad ; 1 \leq r \leq n_1$$
$$h(u_{sr}) = cP_8(m-1+n_1+n_2+\cdots+n_{s-1}+r) - h(u_s)$$
$$s = 2,3,4, \dots, m \quad ; \ r = 1,2,3, \dots, n_s$$

and the induced edge labels are

$$h^+(u_1 u_2) = cP_8(1)$$
$$h^+(u_s u_{s+1}) = h(u_s) + h(u_{s+1}) = cP_8(s) \quad ; \ s = 2,3,4, \cdots m-1$$
$$h^+(u_1 u_{1r}) = cP_8(m-1+r) \quad ; r = 1,2,3, \dots, n_1$$
$$h^+(u_s u_{sr}) = h(u_s) + h(u_{sr}) = cP_8(m-1+n_1+n_2+\cdots+n_{s-1}+r) \ ;$$
$$r = 1,2, \dots, n_s \ ; s = 2,3, \dots, m$$

As a result, the induced edge labels represent the first q Centered Octagonal numbers.

Hence Caterpillars $S(n_1, n_2, n_3, \dots, n_m)$ are COSG for all $n_r \geq 1$

Theorem 2.2

Olive trees are COSG's

Proof

Let u_0 be the root. Let $u_{11}, u_{12}, \cdots u_{1n}$ be the vertices in the first level. Let $u_{22}, u_{23}, \cdots u_{2n}$ be the $n-1$ vertices in the second level and so on. Let u_{nn} be the unique vertex in the n^{th} level. Then $u_0 u_{1r}$ $1 \leq r \leq n$; $u_{1r} u_{2r}$ $2 \leq r \leq n$; $u_{2r} u_{3r}$ $3 \leq r \leq n$; $\cdots u_{n-1 n} u_{nn}$ are the edges in the corresponding levels.

Then G has $\frac{n(n+1)}{2}$ edges and $\frac{n(n+1)}{2} + 1$ vertices

Define $h: V(G) \rightarrow N$ by

$$h(u_0) = 0$$
$$h(u_{1r}) = cP_8(r), \qquad 1 \leq r \leq n$$
$$h(u_{mr}) = cP_8\left[(m-1)n - \frac{m(m-1)}{2} + r\right] - h(u_{m-1,r})$$
$$where\ m(represent\ each\ level) = 2,3 \cdots, n; \ \ r = m, m+1, \cdots, n$$

This induces the edge labels as follows

$$h^+(u_0 u_{1r}) = h(u_0) + h(u_{1r})$$
$$= 0 + cP_8(r)$$
$$= cP_8(r) \qquad 1 \leq r \leq n$$
$$h^+(u_{1r} u_{2r}) = h(u_{1r}) + h(u_{2r})$$

$$= h(u_{1r}) + CP_6[n-1+r] - h(u_{1r})$$
$$= cP_8[n-1+r] \qquad 1 \le r \le n-1$$
$$h^+(u_{2r}u_{3r}) = h(u_{2r}) + h(u_{3r})$$
$$= h(u_{2r}) + P_6[2n-3+k] - h(u_{2r})$$
$$= cP_8[2n-3+r] \qquad 1 \le r \le n-2$$
$$\vdots$$
$$\vdots$$
$$\vdots$$
$$h^+(u_{n-1n}u_{nn}) = h(u_{n-1n}) + h(u_{nn})$$
$$= h(u_{n-1n}) + CP_6\left[(n-1)n - \frac{n(n-1)}{2} + n\right] - h(u_{n-1n})$$
$$= cP_8\left[\frac{n(n+1)}{2}\right]$$

Hence the injective function h induces first $cP_8\left[\frac{n(n+1)}{2}\right]$ edge labels. Therefore, Olive trees are COSG's

Theorem 2.3

Shrub $St(n_1, n_2, \cdots, n_m)$ is a COSG

Proof

Let $V\big(St(n_1, n_2, \cdots, n_m)\big) = \{v, v_r, v_{rs}: 1 \le r \le m, 1 \le s \le n_r\}$ and $E\big(St(n_1, n_2, \cdots, n_m)\big) = \{vv_r, v_r v_{rs}: 1 \le r \le m, 1 \le s \le n_r\}$. Therefore $St(n_1, n_2, \cdots, n_m)$ has $m + n_1 + n_2 + \cdots + n_m + 1$ vertices and $m + n_1 + n_2 + \cdots + n_m$ edges.

Define $h: V\big(St(n_1, n_2, \cdots, n_m)\big) \longrightarrow N$ by

$h(v) = 0$

$h(v_r) = cP_8(r) \qquad 1 \le r \le m$

$h(v_{1s}) = cP_8(m+s) - h(v_1) \qquad 1 \le s \le n_1$

$h(v_{2s}) = cP_8(m+n_1+s) - h(v_2) \qquad 1 \le s \le n_2$

$\vdots$

$h(v_{rs}) = cP_8(m + n_1 + n_2 + \cdots + n_{r-1} + s) - h(v_r) \qquad 1 \le r \le m\,,\ 1 \le s \le n_r$

The induced edge labels are

$$h^+(vv_r) = h(v) + h(v_r) = cP_8(r) \qquad 1 \le r \le m$$
$$h^+(v_1 v_{1s}) = h(v_1) + (v_{1s})$$
$$= h(v_1) + cP_8(m+s) - h(v_1)$$
$$= cP_8(m+s), \qquad 1 \le s \le n_1$$
$$\vdots$$
$$h^+(v_m v_{ms}) = h(v_m) + (v_{ms})$$
$$= h(v_m) + cP_8(m + n_1 + n_2 + \cdots + n_{m-1} + s) - h(v_m)$$
$$= cP_8(m + n_1 + n_2 + \cdots + n_{m-1} + s), \qquad 1 \le s \le n_m$$

Clearly the edges labels are the first $m + n_1 + n_2 + \cdots + n_m$ Centered Octagonal numbers. Hence Shrub $St(n_1, n_2, \cdots, n_m)$ is a COSG.

Theorem 2.4

Banana tree $Bt(n_1, n_2, \cdots, n_m)$ is a COSG

Proof

Let $V\big(Bt(n_1, n_2, \cdots, n_m)\big) = \{v, v_r, w_r, w_{rs}: 1 \le r \le m, 1 \le s \le n_r - 1\}$ $E\big(St(n_1, n_2, \cdots, n_m)\big) = \{vv_r, v_r w_r, v_r v_{rs}: 1 \le r \le m, 1 \le s \le n_r - 1\}$

$Bt(n_1, n_2, \cdots, n_m)$ has $m + n_1 + n_2 + \cdots + n_m + 1$ vertices and $m + n_1 + n_2 + \cdots + n_m$ edges.

Define $h: V\big(Bt(n_1, n_2, \cdots, n_m)\big) \longrightarrow N$ by

$h(v) = 0$
$h(v_r) = cP_8(r) \quad 1 \le r \le m$
$h(w_r) = cP_8(m + r) - h(v_r) \quad 1 \le r \le m$
$h(w_{1s}) = cP_8(2m + s) - h(w_1) \quad 1 \le s \le n_1 - 1$
$h(w_{2s}) = cP_8(2m + n_1 - 1 + s) - h(w_2) \quad 1 \le s \le n_2 - 1$
$h(w_{3s}) = cP_8(2m + n_1 + n_2 - 2 + s) - h(w_3) \quad 1 \le s \le n_3 - 1$
$\vdots$
$h(w_{rs}) = cP_8(2m + n_1 + n_2 + \cdots + n_{r-1} - (r - 1) + s) - h(w_r) \quad 1 \le r \le m,\ 1 \le s \le n_r - 1$

The induced edge labels are

$h^+(vv_r) = h(v) + h(v_r) = cP_8(r), \quad 1 \le r \le m$
$h^+(v_r w_r) = h(v_r) + h(w_r) = cP_8(2r), \quad 1 \le r \le m$
$h^+(w_1 w_{1s}) = h(w_1) + (w_{1s})$
$= h(w_1) + cP_8(2m + s) - h(w_1), \quad 1 \le s \le n_1 - 1$
$= cP_8(2m + s), \quad 1 \le s \le n_1 - 1$
$\vdots$
$h^+(w_m w_{ms}) = h(w_m) + (w_{ms})$
$= h(w_m) + cP_8(m + n_1 + n_2 + \cdots + n_{m-1}(r - 1) + s) - h(w_m)$
$= cP_8(2m + n_1 + n_2 + \cdots + n_{m-1} - (r - 1) + s), \quad 1 \le r \le m, 1 \le s \le n_m - 1$

Clearly the edges labels are the first $m + n_1 + n_2 + \cdots + n_m$ Centered Octagonal numbers. Hence $Bt(n_1, n_2, \cdots, n_m)$ is a COSG

Theorem 2.5

H – graphs are COSG's

Proof

Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices of two copies of the path P_n, respectively. Thus, the two copies have edge sets $\{u_i u_{i+1}: 1 \le i \le n - 1\}$ and $\{v_i v_{i+1}: 1 \le i \le n - 1\}$. Let $u_r v_s$, where $r, s \in \{1, 2, \dots, n\}$, be an edge joining a vertex of the first copy of P_n to a vertex of the second copy. The resulting graph is called an H-graph, which contains $2n$ vertices and $2n - 1$ edges . Define $h: V(H) \longrightarrow N$ by

$h(u_r) = 0$
$h(u_{r-i}) = cP_8(i) - h(u_{r-i+1}) \quad ; \ i = 1,2, \dots, r - 1$
$h(u_{r+i}) = cP_8(r + i - 1) - f(u_{r+i-1}) \quad ; \ i = 1,2,3, \dots, n - r$
$h(v_s) = cP_8(n)$
$h(v_{s-i}) = cP_8(n + i) - h(v_{s-i+1}) \ ; \ i = 1,2, \dots, s - 1$
$h(v_{s+i}) = cP_8(n + s - 1 + i) - h(v_{s+i-1}) \ ; \quad i = 1,2,3, \dots, n - s$

Clearly h is an injective function that establishes

$h^+\big(E(H)\big) = \{cP_8(1), cP_8(2), \dots, cP_8(2n - 1)\}$

This completes the proof.

Theorem 2.6

$H \odot K_1$ graph is a COSG

Proof

Let $u_1, u_2, \dots, u_n$ be the n vertices of the first copy of P_n, and let $v_1, v_2, \dots, v_n$ be the n vertices of the second copy of P_n. For $1 \le i \le n$, let u_i' and v_i' be the pendant vertices adjacent to u_i and v_i, respectively.

The edge sets of the two copies of P_n are $\{u_i u_{i+1}: 1 \le i \le n - 1\}$ and $\{v_i v_{i+1}: 1 \le i \le n - 1\}$.

In addition, let$\{u_i u_i': 1 \le i \le n\}$ and $\{v_i v_i': 1 \le i \le n\}$ be the pendant edges and $u_r v_s$, where $r, s \in \{1, 2, \dots, n\}$, be the connecting edge between the two copies of P_n.

The resulting graph is denoted by $H \odot K_1$. Thus, $H \odot K_1$ has $4n$ vertices and $4n - 1$ edges.

Define $h: V(H \odot K_1) \longrightarrow N$ by

$h(u_r) \quad = 0$

$h(u_{r-i}) = cP_8(i) - h(u_{r-i+1}) \quad ; \; i = 1,2, \dots , r-1$

$h(u_{r+i}) = cP_8(r+i-1) - f(u_{r+i-1}) \; ; \; i = 1,2,3, \dots , n-r$

$h(v_s) \quad = cP_8(n)$

$h(v_{s-i}) = cP_8(n+i) - h(v_{s-i+1}) \; ; \; i = 1,2, \dots , s-1$

$h(v_{s+i}) = cP_8(n+s-1+i) - h(v_{s+i-1}) \; ; \quad i = 1,2,3, \dots , n-s$

$h(u_i') \quad = cP_8(2n-1+i) - h(u_i) \quad ; \quad 1 \leq i \leq n$

$h(v_i') \quad = cP_8(3n-1+i) - h(v_i) \quad ; \quad 1 \leq i \leq n$

Clearly h is an injective function that establishes

$h^+(E(H)) = \{cP_8(1), cP_8(2), \dots , cP_8(4n-1)\}$

This completes the proof.

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Cite this Article:

Sureshkumar, R. ; Maragathavalli, S.; Giridharan, K. (2026). Centered Octagonal Sum Labeling of Some Families of Graphs. Pi International Journal of Mathematical Sciences, 2(4), 19–23.

Journal URL: *https://pijms.com/*

DOI: *https://doi.org/10.59828/pijms.v2i4.44*