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$\mathcal{N}$EUTROSOPHIC $\pi g\gamma^*$ HOMEOMORPHISM IN $\mathcal{N}$EUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the concepts of $\mathcal{N}$eutrosophic $\pi g\gamma^*$ homeomorphism and $\mathcal{N}$eutrosophic M-$\pi g\gamma^*$ homeomorphism in $\mathcal{N}$eutrosophic Topological Spaces. We also analyze $\mathcal{N}$eutrosophic $\pi g\gamma^*$ homeomorphism in comparison with existing $\mathcal{N}$eutrosophic homeomorphisms.

Keywords: *$\mathcal{N}$eutrosophic $\pi g\gamma^*$ continuous mapping, $\mathcal{N}$eutrosophic $\pi g\gamma^*$ closed mapping, $\mathcal{N}$eutrosophic $\pi g\gamma^*$ homeomorphism, $\mathcal{N}$eutrosophic M-$\pi g\gamma^*$ homeomorphism*
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I. INTRODUCTION

Fuzzy set theory, proposed by Zadeh [11] in 1965, provides a mathematical framework in which each element is associated with a membership value between 0 and 1. Building upon this framework, Atanassov K [1] introduced Intuitionistic Fuzzy sets in 1986, incorporating both membership and non-membership degrees to represent uncertainty more comprehensively. Following that, Florentin Samarandache [2] developed $\mathcal{N}eutrosophic$ Set Theory, which further extends these concepts by considering truth, indeterminacy and falsity components independently. This theory was later extended to the topological setting by Salama A A and Albowi S A [10] through the introduction of $\mathcal{N}eutrosophic$ Topological Spaces. Furthermore, Sakthivel K and Manikandan M [6] investigated the concept of Intuitionistic fuzzy $\pi g\gamma^*$ homeomorphisms. Motivated by these developments, the present paper introduces the concept of $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ homeomorphism and $\mathcal{N}eutrosophic$ M-$\pi g\gamma^*$ homeomorphism in $\mathcal{N}eutrosophic$ Topological Spaces.

II. PRELIMINARIES

This section presents the fundamental definitions, operations, and preliminary results concerning $\mathcal{N}eutrosophic$ sets.

Definition 2.1[4]

Let X be a non-empty set and τ_N be the collection of $\mathcal{N}eutrosophic$ subsets of X satisfying the following properties:

- $0_N, 1_N \in \tau_N$
- $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$
- $\cup T_i \in \tau_N$ for every $\{T_i: i \in j\} \subseteq \tau_N$

Then the space (X, τ_N) is called a $\mathcal{N}eutrosophic\ Topological\ Space$ $(\mathcal{NTS})$. The elements of τ_N are called $\mathcal{N}eutrosophic\ \ Open\ Set$ $(\mathcal{NOS})$ and its complement is $\mathcal{N}eutrosophic\ \ Closed\ Set$ $(\mathcal{NCS})$.

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Definition 2.2[4]:

Let (X, τ_N) be a $\mathcal{NTS}$ and $A= \{<x, \mu_A(x), \sigma_A(x), \nu_A(x)>: x \in X\}$ be a $\mathcal{N}eutrosophic\ \mathcal{S}et$ $(\mathcal{NS})$ in X. Then $\mathcal{N}eutrosophic$ closure of A is $N_Cl(A) = \cap \{M:M \text{ is a } \mathcal{NCS} \text{ in X and } A \subseteq M\}$

$\mathcal{N}eutrosophic$ interior of A is $N_Int(A) = \cup \{H:H \text{ is a } \mathcal{NOS} \text{ in X and } H \subseteq A\}$

Definition 2.3[3]:

Let (X, τ_N) be a $\mathcal{NTS}$ and $A= \{<x, \mu_A(x), \sigma_A(x), \nu_A(x)>: x \in X\}$ be a $\mathcal{NS}$ in X. Then A is said to be a $\mathcal{N}eutrosophic$ Generalized closed set (NGCS) if $N_Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\mathcal{NOS}$ in X.

Definition 2.4[7]:

A $\mathcal{N}eutrosophic\ \mathcal{S}et$ A in (X, τ_N) is said to be a $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ closed sets ($N\pi g\gamma^*CS$ in short) if $N_Cl\ (N_Int(A)) \cap N_Int\ (N_Cl\ (A)) \subseteq U$ whenever $A \subseteq U$ and U is $N\pi OS$ in (X, τ_N) and its complement is $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ open sets ($N\pi g\gamma^*OS$ in short)

Definition 2.5:

Let f_N be a mapping from a $\mathcal{NTS}$ (X, τ_N) into a $\mathcal{NTS}$ (Y, σ_N). Then f_N is said to be

- $\mathcal{N}eutrosophic$ Continuous [8] (N Continuous) mapping if $f_N^{-1}(A)$ is a $\mathcal{NCS}$ (X, τ_N) for every $\mathcal{NCS}$ A of (Y, σ_N).
- $\mathcal{N}eutrosophic$ α Continuous [8] (Nα Continuous) mapping if $f_N^{-1}(A)$ is a NαCS (X, τ_N) for every $\mathcal{NCS}$ A of (Y, σ_N).
- $\mathcal{N}eutrosophic$ Generalized Continuous [5] (NG Continuous) mapping if $f_N^{-1}(A)$ is a NGCS (X, τ_N) for every $\mathcal{NCS}$ A of (Y, σ_N).
- $\mathcal{N}eutrosophic$ Closed [5] (N Closed) mapping if $f_N(A)$ is $\mathcal{NCS}$ in (Y, σ_N) for every $\mathcal{NCS}$ A of (X, τ_N).
- $\mathcal{N}eutrosophic$ α Closed [5] (Nα Closed) mapping $f_N(A)$ is NαCS in (Y, σ_N) for every $\mathcal{NCS}$ A of (X, τ_N).
- $\mathcal{N}eutrosophic$ Generalized Closed [9] (NG Closed) mapping if $f_N(A)$ is NGCS in (Y, σ_N) for every $\mathcal{NCS}$ A of (X, τ_N).

Definition 2.6:

Let f_N be a mapping from a $\mathcal{NTS}$ (X, τ_N) into a $\mathcal{NTS}$ (Y, σ_N). Then f_N is said to be

- $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ continuous [8] ($N\pi g\gamma^*$ continuous) mapping if $f_N^{-1}(B)$ is a $N\pi g\gamma^*CS$ (X, τ_N) for every $\mathcal{NCS}$ B of (Y, σ_N).
- $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ Closed [9] ($N\pi g\gamma^*$ Closed) mapping if $f_N(B)$ is $N\pi g\gamma^*CS$ in (Y, σ_N) for every $\mathcal{NCS}$ B of (X, τ_N).
- $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ Open [9] ($N\pi g\gamma^*$ Open) mapping if $f_N(B)$ is $N\pi g\gamma^*OS$ in (Y,σ_N) for every $\mathcal{NOS}$ B of (X, τ_N).
- $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ Irresolute [9] ($N\pi g\gamma^*$ Irresolute) mapping if f_N^{-1} is $N\pi g\gamma^*CS$ in (X, τ_N) for every $N\pi g\gamma^*CS$ B of (Y, σ_N).

Definition 2.7:

A bijection mapping $f_N: (X, \tau_N) \rightarrow (Y,\sigma_N)$ is called a

- $\mathcal{N}eutrosophic$ homeomorphisms [12] if f_N is both a $\mathcal{N}eutrosophic$ continuous mapping and a $\mathcal{N}eutrosophic$ closed mapping.
- $\mathcal{N}eutrosophic$ α homeomorphisms [12] if f_N is both a $\mathcal{N}eutrosophic$ α continuous mapping and a $\mathcal{N}eutrosophic$ α closed mapping.
- $\mathcal{N}eutrosophic$ generalized homeomorphisms [5] if f_N is both a $\mathcal{N}eutrosophic$ generalized continuous mapping and a $\mathcal{N}eutrosophic$ generalized closed mapping.

III. $\mathcal{N}$EUTROSOPHIC $\pi g\gamma^*$ HOMEOMORPHISM

In this section, we have introduced $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ homeomorphisms and studied some of their properties.

Definition 3.1: A bijection mapping $f_N: (X, \tau_N) \rightarrow (Y, \sigma_N)$ is called a $\mathcal{N}eutrosophic$ $\pi g\gamma^*$ homeomorphisms ($N\pi g\gamma^*$ homeomorphisms) if f_N is both a $N\pi g\gamma^*$ continuous mapping and a $N\pi g\gamma^*$ closed mapping.

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.5, 0.5,0.5), (0.4, 0.5, 0.6)\rangle$, $T_2 = \langle y, (0.6,0.5, 0.4), (0.6, 0.5, 0.4)\rangle$. Then $\tau_N = \{0_N, T_1, 1_N\}$ and $\sigma_N = \{0_N, T_2, 1_N\}$ are $\mathcal{NTSS}$ on X and Y respectively. Define a mapping $f_N: (X, \tau_N) \rightarrow (Y, \sigma_N)$ by $f_N(a) = u$ and $f_N(b) = v$. Then f_N is a $N\pi g\gamma^*$ homeomorphisms.

Theorem 3.3: Every $\mathcal{N}eutrosophic$ homeomorphism is a Nπgγ* homeomorphism but not conversely.

Proof:

Let f_N: $(X, \tau_N) \to (Y, \sigma_N)$ be a $\mathcal{N}eutrosophic$ homeomorphism. Then f_N is both a N continuous mapping and a N closed mapping and hence f_N is both a Nπgγ* continuous mapping and a Nπgγ* closed mapping. Therefore, the mapping f_N is a Nπgγ* homeomorphism.

Example 3.4: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.3, 0.3,0.7), (0.2, 0.4, 0.8)\rangle$, $T_2 = \langle y, (0.7, 0.3, 0.2), (0.6, 0.5, 0.3)\rangle$. Then $\tau_N = \{0_N, T_1, 1_N\}$ and $\sigma_N = \{0_N, T_2, 1_N\}$ are $\mathcal{NTSS}$ on X and Y respectively. Define a mapping f_N: $(X, \tau_N) \to (Y, \sigma_N)$ by $f_N(a)$ = u and $f_N(b)$ = v. Then f_N is a Nπgγ* homeomorphism. But f_N is not $\mathcal{N}eutrosophic$ homeomorphism, since f_N is not a N continuous mapping, as $T_2^c = \langle y, (0.2, 0.7, 0.7), (0.3, 0.5, 0.6)\rangle$ is a NCS in (Y, σ_N) but $f_N^{-1}(T_2^c)$ is not a $\mathcal{NCS}$ in (X, τ_N), as the only $\mathcal{NCS}$s in (X, τ_N) are 0_N^c, 1_N^c and T_1^c.

Theorem 3.5: Every $\mathcal{N}eutrosophic$ α homeomorphism is a Nπgγ* homeomorphism but not conversely in general.

Proof:

Let f_N: $(X, \tau_N) \to (Y, \sigma_N)$ be a $\mathcal{N}eutrosophic$ α homeomorphism. Then f_N is both a Nα continuous mapping and a Nα closed mapping. As every Nα continuous mapping is a Nπgγ* continuous mapping and every Nα closed mapping is a Nπgγ* closed mapping, f_N is a Nπgγ* homeomorphism.

Example 3.6: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.4, 0.3, 0.6), (0.3, 0.4, 0.7)\rangle$ and $T_2 = \langle y, (0.6, 0.2, 0.4), (0.7, 0.6, 0.3)\rangle$. Then $\tau_N = \{0_N, T_1, 1_N\}$ and $\sigma_N = \{0_N, T_2, 1_N\}$ are $\mathcal{NTSS}$ on X and Y respectively. Define a mapping f_N: $(X, \tau_N) \to (Y, \sigma_N)$ by $f_N(a)$ = u and $f_N(b)$ = v. Here f_N is a Nπgγ* homeomorphism. But f_N is not a $\mathcal{N}eutrosophic$ α homeomorphism, since f_N is not a Nα continuous mapping, as $T_2^c = \langle y, (0.4, 0.8, 0.6), (0.3, 0.4, 0.7)\rangle$ is a $\mathcal{NCS}$ in (Y, σ_N) but $f^{-1}(T_2^c) = \langle x, (0.4, 0.8, 0.6), (0.3, 0.4, 0.7)\rangle$ is not a NαCS in (X, τ_N)

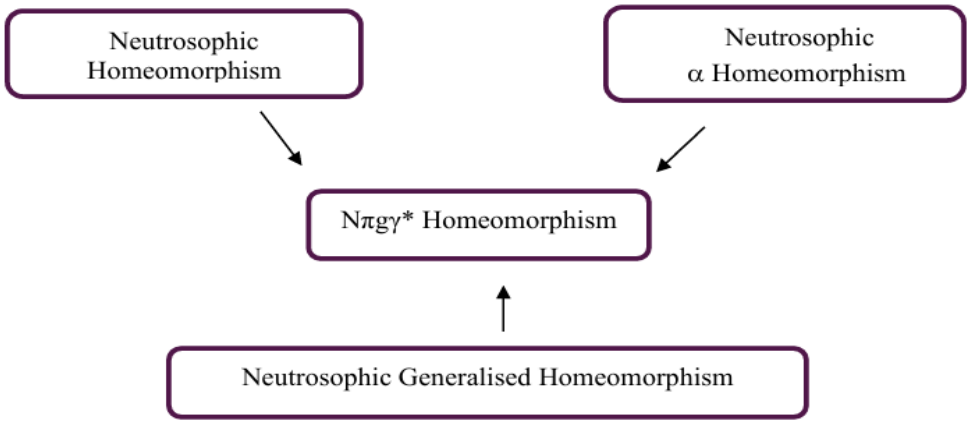
Theorem 3.7: Every $\mathcal{N}eutrosophic$ generalized homeomorphism is a Nπgγ* homeomorphism but not conversely in general.

Proof:

Let f_N: $(X, \tau_N) \to (Y, \sigma_N)$ be a $\mathcal{N}eutrosophic$ generalized homeomorphism. Then f_N is both a $\mathcal{N}eutrosophic$ generalized continuous mapping and a $\mathcal{N}eutrosophic$ eneralized closed mapping. As every $\mathcal{N}eutrosophic$ eneralized continuous mapping is a Nπgγ* continuous mapping and every $\mathcal{N}eutrosophic$ generalized closed mapping is a Nπgγ* closed mapping, f_N is a Nπgγ* homeomorphism.

Example 3.8: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.7, 0.2, 0.1), (0.6, 0.3, 0.2)\rangle$, $T_2 = \langle x, (0.2, 0.3, 0.7), (0.3, 0.4, 0.6)\rangle$. Then $\tau_N = \{0_N, T_1, 1_N\}$ and $\sigma_N = \{0_N, T_2, 1_N\}$ are $\mathcal{NTSS}$ on X and Y respectively. Define a mapping f_N: $(X, \tau_N) \to (Y, \sigma_N)$ by $f_N(a)$ = u and $f_N(b)$ = v. Here f_N is a Nπgγ* homeomorphism but not a $\mathcal{N}eutrosophic$ generalized homeomorphism, since f_N is not a $\mathcal{N}eutrosophic$ generalized continuous mapping, as $T_2^c = \langle y, (0.7, 0.7, 0.2), (0.6, 0.6, 0.3)\rangle$ is a $\mathcal{NCS}$ in (Y, σ_N) but $f_N^{-1}(T_2^c)) = \langle x, (0.7, 0.7, 0.2), (0.6, 0.6, 0.3)\rangle$ is not a NGCS in (X, τ_N).

The relationship between different types of $\mathcal{N}eutrosophic$ homeomorphism is given in the following diagram.
In general, the reverse implications in the above diagram do not hold.



Theorem 3.9: Let $f_N: (X, \tau_N) \to (Y, \sigma_N)$ be a bijective mapping. Then the following are equivalent:

(a) f_N is a Nπgγ* closed mapping,

(b) f_N^{-1} is a Nπgγ* continuous mapping,

(c) f_N is a Nπgγ* open mapping.

Proof:

(a) ⇒ (b) Let B be an $\mathcal{NCS}$ in (X, τ_N). Since f_N is a Nπgγ* closed mapping

$f_N(B) = (f_N^{-1})^{-1}(B)$ is a Nπgγ*CS in (Y, σ_N). This implies f_N^{-1} is a Nπgγ* continuous mapping.

(b) ⇒ (c) Let A be a $\mathcal{NOS}$ in (X, τ_N). Then by hypothesis $(f_N^{-1})^{-1}(A) = f_N(A)$ is a Nπgγ*OS in (Y, σ_N). That is f_N is a Nπgγ* open mapping.

(c) ⇒ (a) Let f_N be a Nπgγ* open mapping. Let A be a $\mathcal{NCS}$ in (X, τ_N). Then A^c is an $\mathcal{NOS}$ in (X, τ_N). By hypothesis $f_N(A^c) = f_N(A)^c$ is a Nπgγ*OS in (Y, σ_N) as f_N is bijective. Therefore $f_N(A)$ is a Nπgγ*CS in (Y, σ_N). Hence f_N is a Nπgγ* closed mapping.

IV. $\mathcal{N}$EUTROSOPHIC M-πgγ* HOMEOMORPHISM

Definition 4.1: A bijective mapping $f_N: (X, \tau_N) \to (Y, \sigma_N)$ is called a $\mathcal{N}eutrosophic$ M-πgγ* (NM-πgγ*) homeomorphism if f_N is both a Nπgγ* irresolute mapping and a Nπgγ* closed mapping.

Example 4.2: Let X = {a, b}, Y = {u, v}, $T_1 = \langle x, (0.4, 0.5, 0.6), (0.3, 0.5, 0.7)\rangle$ and $T_2 = \langle y, (0.8, 0.5, 0.1), (0.7, 0.5, 0.2)\rangle$. Then $\tau_N = \{0_N, T_1, 1_N\}$ and $\sigma_N = \{0_N, T_2, 1_N\}$ are $\mathcal{NTSS}$ on X and Y respectively. Define a mapping $f_N: (X, \tau_N) \to (Y, \sigma_N)$ by $f_N(a) = u$ and $f_N(b) = v$. Here f_N is both a Nπgγ* irresolute mapping and a Nπgγ* closed mapping. Hence f_N is a NM-πgγ* homeomorphism.

Theorem 4.3: Every NM-πgγ* homeomorphism is a Nπgγ* homeomorphism.

Proof:

Let $f_N: (X, \tau_N) \to (Y, \sigma_N)$ be a NM-πgγ* homeomorphism. Therefore, f_N is both a Nπgγ* irresolute mapping and a Nπgγ* closed mapping. Let B be a $\mathcal{NCS}$ in (Y, σ_N). This implies B is a Nπgγ*CS in (Y, σ_N). By hypothesis $f_N^{-1}(B)$ is a Nπgγ*CS in (X, τ_N). Hence f_N is a Nπgγ* continuous mapping. Hence f_N is a Nπgγ* homeomorphism.

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